Chirped W-Shaped Optical Solitons of Modified Nonlinear Schrödinger Equation

Mustafa Mizrak

September 29, 2021

Chirped W-Shaped Optical Solitons of Modified Nonlinear Schrödinger Equation

Mustafa MIZRAK[1]

Department of Computer Engineering, Faculty of Engineering, Sirnak University, Sirnak, Turkey,
mmizrak@sirnak.edu.tr

Abstract. The aim of this paper investigated the exact nonlinearly chirped W-shaped soliton solutions of modified nonlinear Schrödinger equation which is proposed to describe the short pulse propagation in long monomode optical fibres. Firstly, we get the corresponding chirping parameter of the modified nonlinear Schrödinger equation by use of a complex envelope traveling-wave ansatz. Secondly, substituting this chirping parameter the modified nonlinear Schrödinger equation has been reduced to an elliptic differential equation with a fourth-degree nonlinear term. Thirdly, I apply localized soliton ansatz of the sech type which allows for obtaining W-shaped soliton solution. Lastly, we get the 2-dim and 3-dim graphs of the W-shaped soliton solution by giving specific values to the parameters.

Keywords: Ansatz method, W-shaped soliton, Modified nonlinear Schrödinger equation

1 Introduction

The nonlinear Schrödinger (NSL) equation from mid1960's it has been a model for deep water waves, nonlinear optics, nonlinear acoustics and plasma waves. It has been used as a basic identification of pulses on an optical fibre by the early 1970's [1]. Since NSL equation was solved with the Inverse Scattering Transform method by Alexey Shabat and Vladimir Zakharov in 1972, the studies on this equation have been continuing increasingly [2]. Many signals that we encounter in our daily life, such as human speech, animal sounds, radar and mechanical systems, have nonlinear and non-stationary properties, that is, signals whose frequency content changes over time. In general, these signals are often called nonlinear chirp signals [3]. Chirp, such as spread spectrum communications and radar and sonar being characterized in some devices. Recently, there has been much interest in the propagation of nonlinear chirping solitons. Mainly, propagation of an optical pulse described by NSL equation.

The modified nonlinear Schrödinger (MNLS) equation, in which the nonlinear terms are consist of a cubic term and a derivative cubic term, has been proposed to describe the short pulse in long monomode optical fibres: where $q \equiv q(x,t)$ is the complex envelope of the wave,μ and δ are reals and $i^2 = -1$ and the term

$2i\delta\left(|q|^2q\right)_x$ is called the self-steepening term, which causes an optical pulse to become asymmetric and steepen up at the trailing edge. This equation is also called the mixed nonlinear Schrödinger- Derivative nonlinear Schrödinger (NLS-DNLS) equation since Eq. (1) reduces into the standard nonlinear Schrödinger equation (NLS): $iq_t+q_{xx}+2\mu|q|^2q=0$ when $\delta=0$, and the derivative nonlinear Schrödinger equation (DNSL): $iq_t+q_{xx}+2i\delta\left(|q|^2q\right)_x=0$ when $\mu=0$ [4, 12].

2 Mathematical analysis

Our interest is to find exact chirped soliton solutions of the MNLS equation. So, we try to find traveling-wave solutions of Eq. (1) in the form [13, 17]

$$q(x,t)=\rho(\xi)e^{i[\chi(\xi)-\omega t]}, \tag{1}$$

where $\xi=kx-vt$, $\rho=\rho(\xi)$ is the amplitude function and $\chi=\chi(\xi)$ is the phase function. Also, wave velocity and the frequency of wave oscillation are v and ω respectively. The corresponding chirp is given by

$$\delta\omega(x,t)=-\frac{\partial}{\partial t}[\chi(\xi)-\omega t]=-\chi'(\xi). \tag{2}$$

Inserting (2) into (1), the real and imaginary parts yields a pair of relations in two dependent variable ρ and χ.Real part gives

$$2\mu\rho^3+\rho\omega+v\rho\chi'-k^2\rho(\chi')^2-2\delta k\rho^3\chi'+k^2\rho''=0, \tag{3}$$

while imaginary part yields

$$2k^2\rho'\chi'+k^2\rho\chi''-v\rho'+6\delta k\rho^2\rho'=0, \tag{4}$$

where primes denote differentiations with respect to ξ. Multiplying Eq. (5) by ρ and once as a result of integration gives

$$\chi'=\frac{v}{2k^2}+\frac{A}{k^2\rho^2}-\frac{3\delta\rho^2}{2k}, \tag{5}$$

where A is an integration constant. Therefore, the resultant chirp consisting of linear and nonlinear terms is obtained by

$$\delta\omega=-\frac{v}{2k^2}-\frac{A}{k^2\rho^2}+\frac{3\delta\rho^2}{2k}, \tag{6}$$

which shows that the chirping has two intensity dependent chirping terms apart from the linear term [where $I=|q|^2=\rho^2$ being intensity]. On substituting Eq. (6) in Eq. (4) gives

$$k^2\rho''-\frac{A^2}{k^2\rho^3}+\left(\frac{v^2}{4k^2}+\frac{A\delta}{k}+\omega\right)\rho+\left(-\frac{v\delta}{k}+2\mu\right)\rho^3+\left(\frac{3}{4}\delta^2\right)\rho^5=0. \tag{7}$$

After multiplying Eq. (8) by ρ' and integrating with respect to ξ leads to

$$(\rho')^2 = \frac{2B}{k^2} - \frac{A^2}{k^4\rho^2} - \left(\frac{v^2}{4k^4} + \frac{A\delta}{k^3} + \frac{\omega}{k^2}\right)\rho^2 + \left(\frac{v\delta}{2k^3} - \frac{\mu}{k^2}\right)\rho^4 - \frac{\delta^2}{4k^2}\rho^6, \quad (8)$$

where B is the second integration constant. Eq. (9) is a nonlinear differential equation that completes the evolution of the wave amplitude in a nonlinear medium that is governed by Eq. (1). Next, we find chirped soliton solutions of the Eq. (1) for different parameter conditions. Before discussing exact solutions of the Eq. (9), we make the change of variable for the field amplitude

$$\rho^2(\xi) = U(\xi), \quad (9)$$

converts Eq. (9) into the following new auxiliary elliptic equation [18, 22] :

$$(U\prime)^2 = -a_0 + a_1 U + a_2 U^2 + a_3 U^3 - a_4 U, \quad (10)$$

where

$$a_0 = \frac{4A^2}{k^4}, a_1 = \frac{8B}{k^2}, a_2 = -\frac{v^2}{k^4} + \frac{4A\delta}{k^3} - \frac{4\omega}{k^2}, a_3 = \frac{2v\delta}{k^3} - \frac{4\mu}{k^2}, a_4 = \frac{\delta^2}{k^2}. \quad (11)$$

It is known that the elliptic Eq. (11) accepts various solution types such as bright, dark, kink, periodic and solitary wave.

3 Exact Chirped W-Shaped Solitons

To find the nonlinear equation for the wave amplitude, we look for this type of localized soliton ansatz:

$$U(s) = \beta + sech(\mu s), \quad (12)$$

which allows for W-shaped solitons if the unknown parameters β and g satisfy the conditions $\beta g < 0$ and $|g| > \beta$ [23]. Eq. (13) is placed in Eq. (11) and equalizing sech functions of different powers, we get the following system of equations:

$$-a_0 + \beta a_1 + \beta^2 a_2 + \beta^3 a_3 - \beta^4 a_4 = 0, \quad (13)$$

$$g\left(a_1 + 2\beta a_2 + 3\beta^2 a_3 - 4\beta^3 a_4\right) = 0, \quad (14)$$

$$g^2\left(a_2 + 3\beta a_3 - 6\beta^2 a_4 - \mu^2\right) = 0, \quad (15)$$

$$g^3\left(a_3 - 4\beta a_4\right) = 0, \quad (16)$$

$$g^2\left(\mu^2 - a_4\rho^2\right) = 0. \quad (17)$$

Solving these equations, we get

$$\beta = \frac{a_3}{4a_4}, \quad (18)$$

$$g = \pm\left[\frac{3a_3^2 + 8a_2 a_4}{8a_4}\right]^{1/2}, \quad (19)$$

$$\mu = \left[\frac{3a_3^2 + 8a_2 a_4}{8a_4}\right]^{1/2}, \tag{20}$$

and

$$a_0 = -\frac{a_3^2\left(a_3^2 + 16a_3 a_4\right)}{256a_4^3}, a_1 = -\frac{a_3\left(4a_2 a_4 + a_3^2\right)}{8a_4^2}, \tag{21}$$

provided that $a_4 > 0$ and $3a_3^2 + 8a_2 a_4 > 0$. Using Eq. (22), the integral constants A and B are found as $A^2 = a_0 k^4/4$,$B = a_1 k^2/8$. Also, from Eq. (19) and Eq. (20), we can determine the amplitude g in terms of soliton parameter β as

$$g = \pm\sqrt{6\beta^2 + \lambda^2}, \tag{22}$$

where $\lambda^2 = a_2/a_4$. Using of these findings, we can write the exact chirped soliton solution on a continuous- wave (cw) background of Eq. (1) as

$$(x, t) = \left[\beta \pm \sqrt{6\beta^2 + \lambda^2} sech\left[\mu\left(kx - vt\right)\right]\right]^{1/2} e^{i\left[\chi(kx-vt)-\omega t\right]}, \tag{23}$$

The corresponding chirping is given by

$$\delta\omega = -\frac{v}{2k^2} - \frac{A}{k^2\left(\beta \pm \sqrt{6\beta^2 + \lambda^2} sech\left[\mu\left(kx - vt\right)\right]\right)} + \frac{3\delta\left(\beta \pm \sqrt{6\beta^2 + \lambda^2} sech\left[\mu\left(kx - vt\right)\right]\right)}{2k}, \tag{24}$$

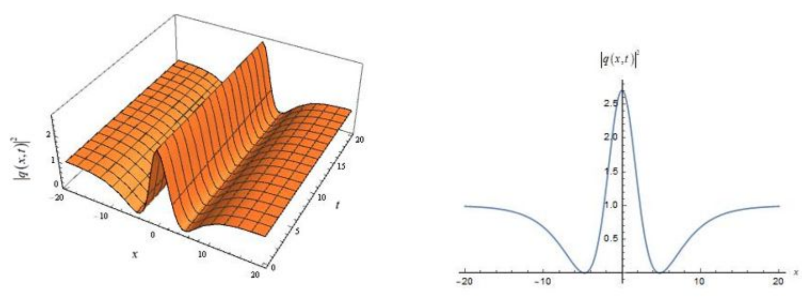

Fig. 1. For the values $\beta = -1, \lambda = 1, \mu = 0.34$ and $v = 0.002$, the intensity $|q(x,t)|^2$ evolution of the W-shaped soliton.

Actually, Eq. (24) explains the evolution of two different types of soliton pulses on a background for the MNLS equation. The parameter β (determined by Eq. (23)) determines the strength of the background in these propagating envelope solutions. Only one of the soliton solutions given in Eq. (24) with the sign − taken into account. The evolution of the soliton intensity calculated from

Eq. (24) is shown in Fig. 1. It receives a W shape of the soliton intensity profile as shown in Fig. 1 and a substantial propagation distance can be observed to remain unchanged throughout. This distinctive soliton shape is due to the self-steepening effect [21]. Also, the + signed soliton solution of Eq. (24) corresponds to a bright pulse solution on the cw background for the MNLS equation.

Conclusion

We have achieved chirped MNLS soliton solutions to equations in this work. To derive a nonlinear differential equation that describes the evolution of the wave amplitude in a system applying the paired amplitude-phase method. It has been shown that the wave amplitude provides a nonlinear differential equation containing two integration constants that can be easily determined by the initial parameters of the wave. So, we get the exact W-shaped soliton solution with nonlinear chirp for Eq. (1). Such chirped soliton solution exists because of the balance between group velocity dispersion and self-steepening effects.

References

1. Scott A.: The Development of Nonlinear Science. October 2004 ChemInform 37(13). doi:10.1393/ncr/i2005-10001-3.
2. Zakharov V.E., Shabat A.B.: Exact theory of two-dimensional self-modulation of waves in nonlinear media. Soviet Phys. JETP, 34 (1972), pp. 62-69.
3. Chen S., Dong X., Peng Z., Zhang W., Meng G.: Nonlinear Chirp Mode Decomposition: A Variational Method. IEEE TRANSACTIONS ON SIGNAL PROCESSING. Advance online publication. doi: 10.1109/TSP.2017.2731300.
4. Zayed E.M.E., Alngar M.E.M.: Application of newly proposed sub-ODE method to locate chirped optical solitons to Triki-Biswas equation. Optik - Int. J. for Light and Electron Optics , 207, (2020) 164360.
5. Zulfiqar A.,Ahmad J.: Soliton Solutions of Fractional Modified Unstable Schrödinger Equation Using Exp-Function Method. Results in Phys., 19, 2020, 103476.
6. Mihalache D., et al.: Analytic method for solving the modified nonlinear Schrodinger equation describing soliton propagation along optical fibers. Phys. Rev. A ,Volume 47, Number 4 April 1993.
7. Mibaile J., and et al.: Chirped solitons in derivative nonlinear Schrödinger equation. Chaos, Solitons and Fractals, 2018, 107, pp. 49-54.
8. Ohkuma K., and et al.: Soliton propagation along optical fibers. Optics Lett., Vol. 12, No. 7 / July 1987.
9. Chen Z.Y., Huang N.N.: Method of Meromorphic Matrix of Transformation For Giving Soliton Solutions of The Modified Nonlinear Schrodinger Equation. Phys. Lett. A, Volume 142, Number 1, 27 November 1989.
10. Doktorov E. V., Shchesnovich V. S.: Modified nonlinear Schrödinger equation: Spectral transform and N-soliton solution. J. of Math. Phys., 36, 7009 (1995).
11. Ding Q., Zhu Z.: On the gauge equivalent structure of the modified nonlinear Schrödinger equation. Phys. Lett. A, 295, 2002, pp.192-197.
12. Wang H.T., and et al.: Modulational instability, interactions of localized wave structures and dynamics in the modified self-steepening nonlinear Schrödinger equation. Wave Motion,91, 2019, 10239.

13. Serge D. Y., and et al.: Optical chirped soliton in metamaterials. Nonlin. Dyn., 90, 2017, pp.13-18.
14. Triki H., and et al.: New envelope solitons for Gerdjikov-Ivanov model in nonlinear fiber optics. Superlattices and Microstructures, 111, 2017, pp.326-334.
15. Biswas A., and et al.: Sub-pico-second chirped optical solitons in mono-mode fibers with Kaup-Newell equation by extended trial function method. Optik ,168, 2018, pp. 208-216.
16. Daoui A.K., and et al.: Propagation of chirped gray optical dips in nonlinear metamaterials, Optics Comm., 430, 2019, pp. 461-466.
17. Triki H., and et al.: Chirped envelope optical solitons for Kaup-Newell equation. Optik- Int. J. for Light and Electron Optics, 177,2019, pp. 1-7.
18. Zayed E.M.E., and et al.: Chirped and chirp-free optical solitons in fiber Bragg grating having dispersive reflectivity with polynomial form of nonlinearity using sub-ODE approach. Optik, 204 , 2020, 164096.
19. Xu G.: Extended auxiliary equation method and its applications to three generalized NLS equations. Abst. Appl. Anal., 2014, Article ID 541370, 7 pp.
20. Ali A.T.: New generalized Jacobi elliptic function rational expansion method. J. Comput. Appl. Math., 235, 2011, pp. 4117-4127.
21. Li-Hua Z., Jin-Yu H.: Sub-ODE's new solutions and their applications to two nonlinear partial differential equations with higher-order nonlinear terms. Commun. Theor. Phys., 52, 2009, pp.773-778.
22. Zeng X., Yong X.: A new mapping method and its applications to nonlinear partial differential equations. Phys. Lett. A, 372, 2008, pp. 6602-6607.
23. Li Z., Li L., Tian H., Zhou G.: New types of solitary wave solutions for the higher order nonlinear Schrödinger equation. Phys. Rev. Lett. 84 (18) (2000) 4096-4099.