A Queue System to the Pandemic Period Length Study

Manuel Alberto M. Ferreira

May 29, 2022

A QUEUE SYSTEM TO THE PANDEMIC PERIOD LENGTH STUDY

Manuel Alberto M. Ferreira

Instituto Universitário de Lisboa (ISCTE-IUL), ISTAR-IUL, BRU-IUL Lisboa, Portugal

Abstract

Epidemics still happen, being the annual influenza outbreaks examples of such occurrences. To estimate the epidemic period length is imperative because, in this period, it is necessary to strengthen the health care, through extra availability in human and material resources. So a huge increase of expenses occurs. As a pandemic is an epidemic with a great population and geographical dissemination, more appropriately this happens with the pandemic period. Mainly using results on the $M|G|\infty$ queue busy period**,** it is presented an application of this queue system to the pandemic period's parameters and distribution function study**.**

Keywords: M|G|∞, busy period, pandemic, cost

JEL Classification: C18

1 The model

All through any queue system operation, a busy period is a period that begins when a costumer arrives at the system finding it empty, ends when a costumer abandons the system letting it empty and there is always at least one customer present.

A M|G|∞ queue system is characterized by the customers arrive according to a Poisson process at rate λ, receive, upon the arrival, a service which time length is a positive random variable with distribution function $G(.)$ and mean α. The traffic intensity is $\rho = \lambda\alpha$.

A pandemic is an epidemic of infectious disease that is spreading through human populations across a large region[1]. Then the $M|G|\infty$ queue can be applied to the pandemic period study, being the parameter λ the rate at which people is infected, supposed the infections occur according to a Poisson process. The service time is the time throughout which an infected person stays sick.

For what interests in this work

---

[1] For instance a continent, or even worldwide, see [10].

-A busy period is a pandemic period.

Another work on this subject is[4]. See also [5].

2 The Pandemic Period

Call PP the random variable pandemic period length. According to the results known for the $M|G|\infty$ queue busy period length distribution

$$E[PP] = \frac{e^{\rho}-1}{\lambda} \qquad (2.1)$$

whatever is an infected person sickness time length distribution, see [9]. As for $Var[PP]$, it depends on the whole sickness time length distribution probabilistic structure. Nonetheless Sathe, see [8], established that

$$\lambda^{-2}\max[e^{2\rho} + e^{\rho}\rho^{2}\gamma_s^2 - 2\rho e^{\rho} - 1; 0] \leq Var[PP] \leq \lambda^{-2}$$
$$[2e^{\rho}(\gamma_s^2 + 1)(e^{\rho} - 1 - \rho) - (e^{\rho} - 1)^2] \qquad (2.2),$$

where γ_s is the sickness time length coefficient of variation.

For α and ρ great enough (very intense infectious conditions) since $G(.)$ is such that for α great enough $G(t) \cong 0, t \geq 0$, the PP distribution function fulfills

$$PP(t) \cong 1 - e^{-\lambda e^{-\rho} t}, t \geq 0 \qquad (2.3),$$

see [7].

Calling N_{PP} the mean number of sick people in the pandemic period, if $G(.)$ is exponential

$$N_{PP} = e^{\rho} \qquad (2.4).$$

For any other $G(.)$ probability distribution

$$N_{PP} \cong \frac{e^{\rho(\gamma_s^2+1)}(\rho(\gamma_s^2 + 1) + 1) + \rho(\gamma_s^2 + 1) - 1}{2\rho(\gamma_s^2 + 1)} \qquad (2.5),$$

see [6]. Of course, multiplying (2.4) or (2.5), as appropriate, by the mean cost of each sick person treatment it is possible to estimate the mean health care cost owing to the pandemic period.

3 Mean Number of Pandemic Periods occurrence in a Time Interval

After the renewal processes theory, see [1], calling $R(t)$the mean number of pandemic periods that begin in $[0, t]$, being $t = 0$ the beginning instant of a pandemic period, it is possible to obtain, see [2,3],

$$R(t) = e^{-\lambda\int_0^t[1-G(v)]dv} + \lambda\int_0^t e^{-\lambda\int_0^u[1-G(v)]dv}du \qquad (3.1)$$

and, consequently,

$$e^{-\rho}(1+\lambda t) \leq R(t) \leq 1+\lambda t \qquad (3.2),$$

see [2]. If the sickness time length is exponentially distributed

$$e^{-\rho\left(1-e^{-\frac{t}{\alpha}}\right)} + \lambda e^{-\rho}t \leq R(t) \leq e^{-\rho\left(1-e^{-\frac{t}{\alpha}}\right)} + \lambda t \quad (3.3)$$

4 Conclusions

To apply this model it is necessary the infections occur according to a Poisson process at constant rate. It is a hypothesis perfectly admissible in this kind of phenomena, since they have great geographic spread, and it is considered the mean arrival rate for the pandemic period as the constant rate.

Among the results presented, (2.1), (2.2), (2.5) and (3.2) are remarkable for the easiness and also for requiring only the knowledge of the infectious rate λ, the mean sickness time α, and the sickness time variance. The other results are more complex and demand the goodness of fit test for the distributions indicated to the sickness times.

References

[1] E. Çinlar, Introduction to stochastic processes, Prentice Hall Inc., New Jersey, 1975.

[2] M. A. M. Ferreira, M|G|∞ queue busy cycle renewal function for some particular service time distributions, Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XII), Virt, Slovakia, 2005, 42-47. Accession Number: WOS: 000288041600006.

[3] M. A. M. Ferreira, Differential equations important in the M|G|∞ queue system transient behaviour and busy period, Proceedings of the APLIMAT 2005-4th International Conference, Bratislava, Slovakia, 2005, 119-132. Accession Number: WOS: 000273992100006.

[4] M. A. M. Ferreira, The pandemic period length modelled through M|G|∞ queue systems, Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XVII),Virt, Slovakia, 2014, 43-47.Accession Number: WOS:000350272000007.

[5] Ferreira, M. A. M., Some achievements from a statistical queuing theory research. Acta Scientiae et Intellectus, **2**(2016), 1, 70-83.

[6] M. A. M. Ferreira and J. A. Filipe, Economic crisis: using M|G|∞ queue systems busy period to solve logistics problems in an organization, China-USA Business Review, **9** (2010), 9, 59-63.

[7] M. A. M. Ferreira, J. A. Filipe and M. Coelho, Unemployment modelled through M|G|∞ systems, International Journal of Latest Trends in Finance and Economic Sciences, **4**(2014), 1, 677-679.

[8] Y. S. Sathe, Improved bounds for the variance of the busy period of an M|G|∞ queue, Advances in Applied Probability, **17** (1985), 4, 913-914. http://dx.doi.org/10.2307/1427096

[9] L. Takács, An introduction to queueing theory, Oxford University Press, 1962.

[10] http://en.wikipedia.org/wiki/PandemiAccessed 8th March 2018.