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Abstract Group-Buying has becoming a popular commodity trading mode in current business modes. However, the existing unified price of group-buying often determine the price by setting the ladder function according to the final quantities. This method not only ignores the contributions of participants to group-buying, but also cannot overcome the impact of buyers' false reports on group buying mechanism. In this paper, a pricing method of online group-buying based on continuous price function is proposed. We adopt an algorithm called VCG4GB, buyers' payments are the sum of commodities price and the extra amount by purchase quantity. The mechanism motivates buyers to report truthful preference through the compensatory payment. We prove that the mechanism has economic attributes such as incentive compatibility through theoretical proof and simulation experiments.

Key words: Incentive Compatibility; Group-Buying; VCG; Pricing

1 Introduction

Group-buying is an emerging way to purchase commodities. It is a model with multiple buyers to purchase commodities at a discount price [1]. When the number of buyers reaches a certain amount, the seller can provide products or services at a markedly reduced price. Because of the number of consumers organized by group-buying, seller can sell commodities to consumers at a lower price. Not only the

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buyer can enjoy the lower price in group-buying mechanism, but the seller can benefit from the aggregate demands. In such situation, the utilities of buyers raise as a result of the massive demands and it ensures the beneficial growth of sellers. It achieves a win-win situation.

Group-buying models can be used to design some effective volume discounts. Some papers have pointed out that a perfect mechanism depends on pricing [2], so this paper is committed to designing a group-buying mechanism based on pricing. We do some researches about group-buying and VCG mechanism. AuctionBot[8] is an auction server where users can apply auctions and sell items. The Kasbah[9] system provides a market where users can set agents and trade goods in agents based on simple protocols. FishMarket[10] provides an electronic auction website where users can give agents a set of compiled bidding strategies. Tete-A-Tete[11] provides an electronic marketplace for cooperation negotiations among agents. However, none of the above systems involves quantity discounts. The article [12] through modeling analysis shows that group-buying auction is weaker than fixed price mechanism. Paper [5] purposes a better mechanism for the shill bids problem in combinatorial auction. However, this mechanism does not satisfy Pareto optimality. The article [6] proposed a stable and effective buyer coalition formation method for the electronic market called "Group Buy Auction". In article [7], an algorithm for the formation and revenue segmentation of combined coalition is proposed. These methods do not take into account the issue of economic efficiency, such as Pareto efficiency or social surplus. Paper [13] considers the participants' externalities and puts forward the online pricing mechanism. Vickrey purposed an counterspeculation mechanism in the paper [4], which is the rudiment of VCG mechanism. The existing unified purchase price group-buying mode determine the sales price according to the total quantity by setting the staircase decline price curve. However, it can be seen that people buy different numbers commodities have different effects on final price, but they get the same price finally. In the ordinary group-buying, buyers have motivation to submit false reports in order to improve their utilities. Therefore, the existing way of group-buying not only ignores the contributions of participants to group-buying, but also cannot overcome the impact of buyers' false reports on group buying mechanism

Therefore we put forward a new group-buying mechanism. Our mechanism will solve the problems mentioned above. First, the mechanism is Pareto's efficiency because our mechanism uses the VCG method[3] which can maximize the total utility. Our mechanism is a truthful mechanism, and buyers are quoted independently, because of the setting of pricing, buyers have no incentive to offer false quotes and the false reports cannot make them achieve higher utilities.

2 Problem description

As shown in Figure 1, seller and buyers submit reports to the platform at the same time. After getting all information, the platform aggregated the information and the

equilibrium price is obtained after the intersection of two curves. Allocation mechanism of the platform decides allocation according to the equilibrium price, and the payment mechanism calculates payments. Finally, the allocation and payment mechanism will return the allocation results and the payment price to participants.
The quantity of goods can be selles is: $Q(Q \approx +\infty)$. Seller submits supply func-

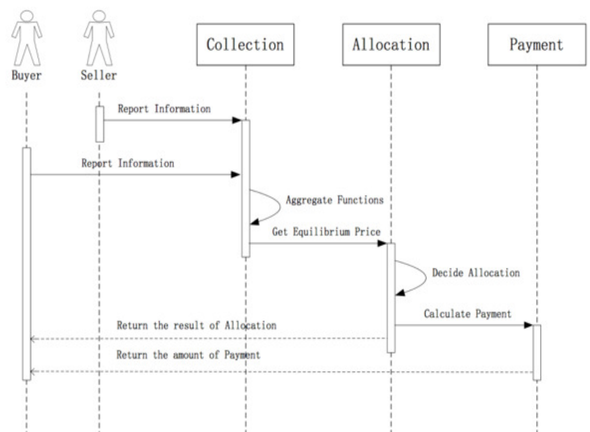

Fig. 1: Time sequence diagram of group buying platform

tion $S(Q)$ to mechanism. Buyers can be denoted as $B = \{b_1, b_2, \ldots, b_n\}$. There are n buyers to take part in the group-buying. Buyer $b_i(b_i \in B)$ submits demand function $D(Q)$ to the mechanism according to his strategy, and gets the true value $v_i(q_i)$ of his expected quantity of merchandise.

Definition 1. Supply function. $S(Q) : Q \to P$ is a continuous curve. It is highest at the beginning and decreases with increased quantities. Until the number reach a minimum, the price will no longer change.

Definition 2. Protective condition. For seller's supply curve, the increased of total volume of goods will cause an increase in the total amount of transaction. Formally, for $q_a > q_b$, $S(q_a) * q_a > S(q_b) * q_b$.

Definition 3. Demand function. $D(Q) : Q \to P$ is a function submitted by buyer to the mechanism. $v_i(q_i)$ represents the value of expected goods of buyer b_i. If obtained goods is less than the expected number, $v_i(q_i) = 0$.

When the mechanism obtains demand function and supply function, the intersection point of the two function curves is the equilibrium price. We use $u_i = v_i(q_i) - m_i$ to expresse buyer b_i's utility $(b_i \in B)$, that is, the value of a certain quantity of merchandise minus this buyer's payment; and the utility of seller is $u_s = \sum_{i \in B} m_i - S(Q) * Q$, that is, the sum of buyers' payment minus the product of group buying price and transaction volume. So, social welfare $SW = \sum_{i \in B} u_i + u_s$ is the social utility from the personal utility function, which is defined as the sum of utility of participants.

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Algorithm 1 embodies the execution flow of this mechanism. First, seller and buyers report their functions to the mechanism respectively. According to Definition 5, by seller's supply curve and buyers' aggregate demand curve, the total amount of transaction can cause the equilibrium price p^* and the total number of transactions Q at this price. The ordering of the buyers is fixed.

We achieve the quantity of goods of buyers according to the current demand curve of other effective buyers when the buyer is not involved and the demand curve of all effective buyers when the buyer is involved and the final group-buying price p^*. The result is added to the distribution result set; the next step is to calculate the buyer's payment amount, and the mechanism ends. The group-buying begins. Seller and buyers report their functions to the mechanism respectively, and the mechanism obtains the total supply function and the aggregate demand function. Get the final group-buying price p^* and final group-buying quantity Q from $S_n - D_n = 0$. Buyer b_i participate in group-buying, and get the number of goods q_i of buyer b_i according to the demand function and the final group-buying price p^*.

The payment of buyer b_i is $m_i = r_i + t_i$,

$$r_i = q_i * p^* \tag{1}$$

$$t_i = p^{*-i} * Q_{-i} - p^* * (Q - q_i) \tag{2}$$

(1): The number of goods q_i buyer b_i obtained at the price p^*.At this point, the volume of transactions purchased by the group is Q and the amount of transactions that other buyers at the price p^* is $p^* * (Q - q_i)$. (2): We assume that buyer b_i do not participate in group-buying, and the final group-buying price is p^{*-i}. At this time, the total transaction volume of other buyers under the price p^{*-i} is $p^{*-i} * Q_{-i}$. The difference between the two results is the impact of b_i buyer's participation on the final price of the group-buying.

With the participation of many buyers, buyers get a lower group-buying price, so that buyer b_i will pay an extra money of his influence for all other buyers. On the other hand, because the demand is not large enough, buyer b_i cannot obtain the lowest group-buying price if only he took part in. So the extra payment also makes up for the price change caused by the shortage of quantity. So t_i is the compensatory payment of buyer b_i.

Algorithm 1: VCG4GB Mechanism

Input: Reports submitted by Buyers: $D_n = \{d_1, \ldots, d_n\}$ and Reports submitted by Seller: S_n

Output: Allocation: $O_n = \{q_1, \ldots, q_n\}$ and Payment: $M_n = \{m_1, \ldots, m_n\}$

for $i = 1 \rightarrow n$ **do**
    initiate $D_n \leftarrow 0$, initiate $O_n \leftarrow \emptyset$;
    if $d_i < S_n$ **then**
        $q_i \leftarrow 0$;
    end
    else
        $D_n \leftarrow D_n + d_i$
    end
end
$Q, p^* \leftarrow S_n - D_n = 0$;
for $i = 1 \rightarrow n$ **do**
    if $q_i = 0$ **then**
        $O_n \leftarrow O_n + \{q_i\}$
    end
    else
        $q_i \leftarrow D_n^{-1}(p^*) - D_{-i}^{-1}(p^*)$;
        $O_n \leftarrow O_n + \{q_i\}$;
    end
end
for $i = 1 \rightarrow n$ **do**
    if $q_i > 0$ **then**
        $Q_-i, p^{*-i} \leftarrow S_n - D_{-i} = 0$;
        $m_i \leftarrow q_i * p^* + p^{*-i} * Q_-i - p^* * (Q - q_i)$;
    end
    else
        $m_i \leftarrow 0$;
    end
    $M_n \leftarrow \{m_i\}$;
end

4 Experiment

4.1 Experiment design

In this paper, in order to compare the advantages and disadvantages of our mechanism, the experimental section compares the VCG4GB mechanism with the other mechanisms. The main idea of SBM4GB(Single Buyer Match for Group Buying)mechanism is that buyers' demand information is independent of each other. DP4GB(Direct Payment for Group Buying)mechanism aggregated the buyers' demand functions with the seller's supply function to get equilibrium price and the total quantity of the transaction commodity. The payment amount in this algorithm does not attach to the compensatory payment and it calculates payments directly according to the equilibrium price and the quantity of goods.

The experimental part of this chapter is implemented in Java and runs on 2.7 GHz Intel Core i5 processor and 8 GB memory local computer. The parameters of this simulation experiment are as follows:

Table 1: simulation experimental parameters

parameter	description	range
B_s	Seller price	[50,150]
Protection conditions	Protection condition of price function	$S(q_i)*q_i < S(q_i+1)*(q_i+1)$
B_n	Number of buyers	[0,50]
B	Buyer's quoted range	[30,180]
B_q	Buyer's demand range	[0,5]
Q	Number of goods prepared by the seller	250
r	Seller reserve pricer	50
V	Buyer valuation	[50,200]
SW	Social welfare	$\sum v_i(q_i) - p^* * Q$

4.2 Experimental analysis

The experimental analysis is listed in Figures 2. In the mechanisms of this chapter,

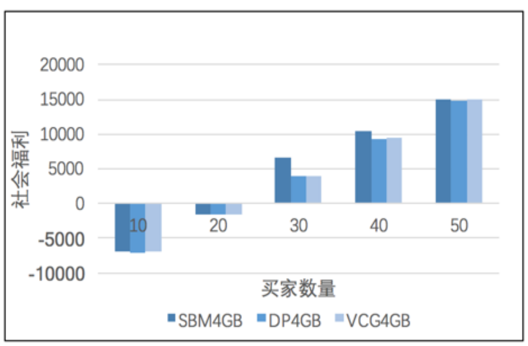

Fig. 2: Social Welfare of three mechanisms

there is a balance between supply and demand between seller and buyers according to the equilibrium price setting. As can be seen from the graph, when there are only 10 buyers and 20 buyers, the social welfare of the three mechanisms is negative, when the number of buyers is too small, and the seller's supply exceeds demand. With the number of buyers increasing to 30 or more, the social benefits of the three mechanisms are increasing. Among them, the SBM4GB mechanism was obviously larger than the other two mechanisms when the 30 people participated. The increase of the mechanism was reduced, and the supply and demand of the goods was balanced to 50, and the social welfare gap between the three mechanisms was not significant. The impact of the number of personal merchandise on the selling price is much less than that of more people participating. In this case, the group-buying is not significant. It can be seen that the VCG4GB mechanism can achieve relatively high social welfare under the condition of satisfying incentive compatibility.

5 Conclusion

In this paper, we adopt the method of group-buying and VCG mechanism to propose a mechanism of allocation and payment based on purchase quantity. This mechanism allocates group-buying products according to the VCG4GB algorithm and calculates payment of buyers. According to the algorithm, the mechanism has the following characteristics: the social welfare of the mechanism is the largest, the protection strategy makes the buyer pay non-negative compensatory payment, the false report will not make buyers' own utility higher, so they will submit truthful reports. This paper considers the situation that the seller has a large number of buyers. In reality, there are also multiple sellers in group-buying. There is no in-depth study on the problem of allocation and payment with multiple sellers and buyers. In the future, the problem of how to allocate the seller's supply besides the problem of the buyer's allocation will be deeply studied.

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