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# Contact Heat Transfer of a Cutting Diamond Disk With a Boundary Layer of Air and a Decrease in the Temperature of Heating of the Disk

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## Abstract.

Cutting of natural and artificial building materials is most often carried out with diamond cutting disks on a metal base at cutting speeds of about 50-80 m / s. The intensity of the cutting process causes a significant heat release, as a result of which the disk temperature rises to unacceptable values. The value of these unacceptable temperatures is about 600 - 650<sup>0</sup>C.

At these temperatures, graphitization of diamond grains occurs, i.e. loss of diamond layer and loss of cutting properties. In addition, a thin diamond disk (thickness 1 - 3 mm) is deformed, which leads to jamming and its tensile strength at these temperatures is reduced by half, which creates the risk of rupture by centrifugal forces.

In this work, it is taken into account that during the rotation of the disk, a boundary layer of air is created around it, which is stationary relative to the disk. Consequently, contact heat transfer occurs between the disk and the boundary layer, and then convective heat transfer occurs between the boundary layer and the surrounding air. This scheme allows you to more accurately determine the time of safe operation of the diamond disk.

Contact heat transfer between the wheel and the boundary layer is not effective enough to lower the temperature.

When air with a negative temperature is introduced into the boundary layer by means of a Rank-Hillsch tube, the disk temperature decreases by about 10%.

When a sprayed coolant (fog cooling) is introduced into the boundary layer by means of an ejector tube, the disk temperature decreases by 25%, which ensures an increase in the time of continuous operation.

**Key words.** Boundary layer, air pressure, boundary layer thickness, cutting zone, mathematical modeling, cooling media, operating time, diamond disc.

## 1. Introduction

Cutting of natural and artificial building materials is most often carried out with diamond cutting disks on a metal base at cutting speeds of about 50-80 m/s. The intensity of the cutting process causes a significant heat release, as a result of which

the disk temperature rises to unacceptable values. The value of these unacceptable temperatures is about 600 - 650°C.

At these temperatures, graphitization of diamond grains occurs, i.e. loss of diamond layer and loss of cutting properties. In addition, a thin diamond disk (thickness 1 – 3 mm) is deformed, which leads to jamming and its tensile strength at these temperatures is reduced by half, which creates the risk of rupture by centrifugal forces.

Thus, the heating temperature of the disk should not exceed 600 °C. Therefore, the operating time of a diamond cutting disc is the time during which it is heated during continuous operation to a temperature of 600 °C. The longer this time, the higher the resistance of the diamond blade. In the present work, mathematical modeling is performed taking into account contact heat transfer between a rotating disk and a boundary layer

The simulation of the process of interaction of the disk with the environment is carried out according to the results of which it is possible to determine the time of disk performance. However, in this paper, convective heat transfer between the disk and the surrounding air is considered at a time when the heat transfer process is more complex. When the disk rotates around it, a boundary layer of air is created that is stationary relative to the disk. Consequently, contact heat transfer occurs between the disk and the boundary layer, and then convective heat transfer occurs between the boundary layer and the surrounding air. This scheme allows you to more accurately determine the time of safe operation of the diamond disk.

## 1 Literature Review

A significant number of works devoted to this subject firstly consider convective heat transfer between the disk and air, moreover, at high Reynolds numbers, which does not correspond to our case. Works [2-14] consider precisely such cases, therefore, the data presented in these works cannot be used in our studies.

### Research Methodology

The purpose of this work is to investigate the contact heat transfer process between a wheel and a boundary layer of air, based on which to determine the possibilities of cooling a rotating wheel by changing the thermophysical characteristics of the boundary layer.

The tasks to be solved in this paper are as follows.

1. Mathematical modeling of the heat transfer process to determine the intensity of the latter.
2. Mathematical modeling of the disk cooling process when changing the thermophysical characteristics of the boundary layer.

Calculations are carried out in accordance with the scheme presented in Fig.1.

Here is a solution for a thin rotating disk heated at the end in the contact area and cooled from the side surfaces as a result of contact heat exchange with the boundary layer. Fig. 1 [15.16].

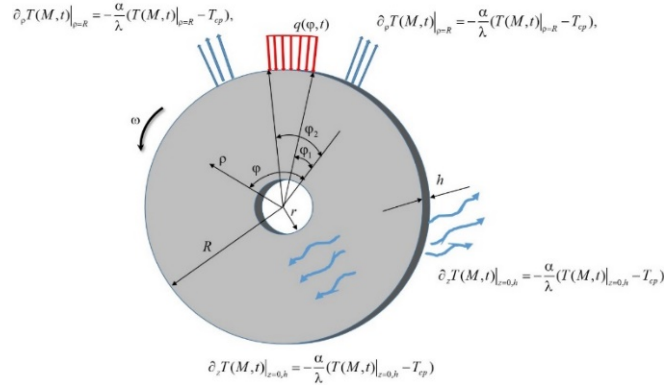


Figure 1. Design scheme for cooling of a rotating disk

A thick disk  $h$  rotates in a plane  $XOY$  with angular velocity  $\omega$ . At the end of the circle, within the limits of the contact arc, a heat source of intensity  $q(\varphi, t)$  is defined, depending on the cutting conditions. On the lateral surfaces of the disk and outside the contact arc, heat transfer occurs at the end according to the Newton-Richmann law, and on the lateral surfaces of the disk heat transfer is considered not with the environment, but with the boundary layer, the temperature of which can be varied over a wide range up to  $-50$  °C.

The boundary-value heat conduction problem for a thin disk in the presence of heat transfer through the side surfaces, taking into account the angular velocity in the polar coordinate system  $(\rho, \varphi)$ , has the form:

$$\partial_t T = a(\partial_r^2 + \rho^{-1}\partial_r + \rho^{-2}\partial_\varphi^2)T + \omega\partial_\varphi T - \frac{2\alpha_*}{\rho cb}(T - T_{bl}), \quad (1)$$

$$\partial_r = \frac{\partial}{\partial r}, \partial_\varphi = \frac{\partial}{\partial \varphi}, \partial_t = \frac{\partial}{\partial t}$$

$$T \equiv T(r, \varphi, t)$$

Initial condition

$$T(\rho, \varphi, z, t)|_{t=0} = T_0 \quad (2)$$

Boundary conditions

$$-\lambda\partial_\rho T(M, t)|_{\rho=R} = \alpha(T(M, t)|_{\rho=R} - T_{cp}), \varphi \notin [\varphi_1, \varphi_2]$$

$$\partial_\rho T(M, t)|_{\rho=R} = -\frac{q(\varphi)}{\lambda}, \varphi \in [\varphi_1, \varphi_2] \quad (3)$$

where  $T_{bl}$  – boundary layer temperature;  $T_0$  – initial disk temperature;  $\alpha_*$  – coefficient of convective heat transfer between rotating disc and boundary layer,  $c$  – specific heat, (J/kg·grad);  $\rho$  – substance density (kg/m<sup>3</sup>)  $\rho c$  – (J/(m<sup>3</sup> · deg);  $\alpha$  – coefficient of heat output;  $\lambda$  – coefficient of thermal conductivity;  $b$  – disk thickness;  $T_{cp}$  – ambient temperature.

Using substitution  $T(M, t) = \theta(M, t) \exp\left(-\frac{\omega\rho}{2a}\varphi - \frac{\omega^2}{4a}t\right)$

The boundary problem (1) - (3) is reduced to the form:

$$\partial_t \theta(M, t) = a(\partial_r^2 + r^{-1}\partial_r + r^{-2}\partial_\varphi^2)\theta(M, t) - \frac{2\alpha_*}{\rho cb}(\theta(M, t) - T_{bl}), M = M(r, \varphi) \quad (4)$$

$$\theta(M, t)|_{t=0} = \theta_0$$

$$\lambda \partial_\rho \theta(M, t) \Big|_{\rho=R} = -\alpha(\theta(M, t) \Big|_{\rho=R} - T_{cp}), \varphi \notin [\varphi_1, \varphi_2]$$

$$\partial_\rho \theta(M, t) \Big|_{\rho=R} = f(\varphi, t), f(\varphi, t) = \frac{q(\varphi)}{\lambda} \partial_\rho \exp\left(-\frac{\omega\rho}{2a}\varphi - \frac{\omega^2}{4a}t\right), \varphi \in [\varphi_1, \varphi_2] \quad (5)$$

In order to avoid the application of the Laplace transform and the difficulties with its treatment, we proceed as follows. We divide the time interval  $T$  into  $M$  intervals of length  $h = TM^{-1}$  and replace the time derivative by the difference relation

$$\partial_t \theta(\rho, \varphi, t) = \frac{\theta_j(\rho, \varphi) - \theta_{j-1}(\rho, \varphi)}{h}, \theta_j(\rho, \varphi) = \theta(\rho, \varphi, jh), j = 1, 2, \dots$$

then

$$(\partial_r^2 + r^{-1}\partial_\rho + r^{-2}\partial_\varphi^2)\theta_j(\rho, \varphi) - \mu^2\theta_j(\rho, \varphi) = F_j(\rho, \varphi) \quad (6)$$

$$\theta_0(\rho, \varphi) = \theta_0$$

$$\lambda \partial_\rho \theta_j(\rho, \varphi) \Big|_{\rho=R} = -\alpha(\theta_j(\rho, \varphi) \Big|_{\rho=R} - T_{cp}), \varphi \notin [\varphi_1, \varphi_2]$$

$$\partial_\rho \theta_j(\rho, \varphi) \Big|_{\rho=R} = f_j(\varphi), f_j(\varphi) = \frac{q(\varphi)}{\lambda} \partial_\rho \exp\left(-\frac{\omega\rho}{2a}\varphi - \frac{\omega^2}{4a}jh\right), \varphi \in [\varphi_1, \varphi_2] \quad (7)$$

where

$$\mu^2 = \frac{2\alpha_*}{\rho c b a} + \frac{1}{ah}, F_j(\rho, \varphi) = T_{bt} - (ah)^{-1}\theta_{j-1}(\rho, \varphi)$$

Let us construct a discontinuous solution [16] of the heat equation for an unbounded plane  $0 \leq \rho < \infty, |\varphi| < \pi$  containing a circular defect occupying the region  $r = R, -\pi \leq \varphi \leq \pi$ , upon transition through which they suffer discontinuities of continuity of the first kind, the temperature  $\theta_j(\rho, \varphi)$  and the heat flux  $\partial_\rho \theta_j(\rho, \varphi)$  with given jumps.

$$\theta_j(R+0, \varphi) - \theta(R-0, \varphi) = \langle \theta_j(R, \varphi) \rangle \quad (8)$$

$$\partial_\rho \theta(R+0, \varphi) - \partial_\rho \theta(R-0, \varphi) = \langle \partial \theta_j(R, \varphi) \rangle$$

those, a solution that satisfies the heat equation everywhere except for defect points. At these points, jumps in temperature and heat flux are set.

To construct a discontinuous solution of equation (6) with jumps (8), we apply the finite Fourier transform in the variable  $\varphi$ :

$$\theta_j(r, \varphi) = \sum_{n=-\infty}^{\infty} \theta_{j,n}(\rho) e^{-in\varphi}, \theta_{j,n}(\rho) = \Phi_n[\theta_j] \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} \theta_j(r, \varphi) e^{in\varphi} d\varphi, j = 1, 2, \dots$$

and, Hankel transform, according to a generalized scheme [16]:

$$\theta_{j,n,\alpha}^j = \int_0^{\infty} \rho \theta_{j,n}(\rho) J_n(\alpha\rho) d\rho = \left( \int_0^{R-0} + \int_{R+0}^{\infty} \right) \rho \theta_{j,n}(\rho) J_n(\alpha\rho) d\rho$$

where  $J_n(z)$  – Bessel function.

As a result, we obtain:

$$\theta_{n,\alpha}^j = \frac{R}{\alpha^2 + \mu^2} \{J_n(\alpha R) \langle \partial \theta_n^j(R) \rangle - \langle \theta_n^j(R) \rangle \partial_R J_n(\alpha R)\} + \frac{F_{n,\alpha}^j}{\alpha^2 + \mu^2}$$

Where  $F_{n,\alpha}^j = \int_0^{\infty} \rho F_{j,n}(\rho) J_n(\alpha\rho) d\rho, F_{j,n} = \Phi_n[F_j]$

After reversing the Hankel transform, the required discontinuous solution of the heat equation in Fourier transforms can be written as:

$$\theta_{j,n}(\rho) = R \{ \langle \partial \theta_{j,n}(R) \rangle G_n(\rho, R) - \langle \theta_{j,n}(R) \rangle \partial_\rho G_n(\rho, R) \} + F_{j,n},$$

$$G_n(r, R) = \int_0^{\infty} \frac{\alpha J_n(\alpha\rho) J_n(\alpha R)}{\alpha^2 + \mu^2} d\alpha, F_{j,n}(\rho) = \int_0^{\infty} \frac{\alpha J_n(\alpha\rho) F_{n,\alpha}^j}{\alpha^2 + \mu^2} d\alpha \quad (9)$$

Taking into account that:

$$\{(\theta_{n,\lambda}^j(R)), \langle \partial \theta_{n,\lambda}^j(R) \rangle\} \equiv \frac{1}{2\pi} \int_{-\pi}^{\pi} \{(\theta_{j,n}(R, \psi)), \langle \partial \theta_{j,n}(R, \psi) \rangle\} e^{-in\psi} d\psi,$$

We write the discontinuous solution (9) in the form:

$$\theta_{n,\lambda}^j(r, \varphi) = \frac{R}{2\pi} \left\{ \int_{-\pi}^{\pi} \chi_j^{1,-}(\psi) G_n(\rho, R) e^{-in\psi} d\psi - \int_{-\pi}^{\pi} \chi_j^{2,-}(\psi) \partial_\rho G_n(\rho, R) e^{-in\psi} d\psi \right\} + F_{j,n}(\rho),$$

$$\chi_j^{1,-}(\psi) = \langle \theta_{j,n}(R, \psi) \rangle, \chi_j^{2,-}(\psi) = \langle \partial \theta_{j,n}(R, \psi) \rangle$$

Using the inverse finite Fourier transform formula, by  $\varphi$ , and the addition theorem [16].

$$\sum_{n=-\infty}^{\infty} J_n(\alpha r) J_n(\alpha R) e^{in\phi} = J_0(\alpha, r_*), r_*^2 = r^2 + R^2 - 2rR \cos \phi$$

we obtain

$$\theta_j(\rho, \varphi) = \frac{R}{2\pi} \left\{ \int_{-\pi}^{\pi} \chi_j^{2,-}(\psi) K(\rho, R, \varphi - \psi) d\psi - \partial_\rho \int_{-\pi}^{\pi} \chi_j^{1,-}(\psi) K(\rho, R, \varphi - \psi) d\psi \right\} + F_j(\rho, \varphi),$$

$$K(\rho, R, \phi) = \sum_{n=-\infty}^{\infty} G_n(r, R) e^{in\phi} = \int_0^{\infty} \frac{\alpha}{\alpha^2 + \mu^2} \sum_{n=-\infty}^{\infty} J_n(\alpha r) J_n(\alpha R) e^{in\phi} d\alpha$$

$$= \int_0^{\infty} \frac{\alpha J_0(\alpha, r_*)}{\alpha^2 + \mu^2} d\alpha$$

If we use the formula 6.532 (17) from [3], we obtain

$$K(r, R, \phi) = \int_0^{\infty} \frac{\alpha J_0(\alpha, r_*)}{\alpha^2 + \mu^2} d\alpha = K_0(r_* \mu)$$

Then:

$$\theta_j(\rho, \varphi) = \frac{R}{2\pi} \left\{ \int_{-\pi}^{\pi} \chi_j^{2,-}(\psi) K_0(r_* \mu) d\psi - \partial_\rho \int_{-\pi}^{\pi} \chi_j^{1,-}(\psi) K_0(r_* \mu) d\psi \right\} + F_j(\rho, \varphi), \quad (10)$$

We illustrate the technique of using discontinuous solutions to solve the boundary value problem. This technique is based on the idea that the boundary of the disk  $\rho = R$  be considered a defect. We consider the third boundary value problem:

$$\theta_j(R - 0, \varphi) + \kappa \partial_\rho \theta_j(R - 0, \varphi) = f_j(\varphi) \quad (11)$$

Given that outside the domain specified in the conditions of the problem, the solution is identical to zero, i.e. at  $\rho > R$ ,  $\theta_j(\rho, \varphi) \equiv 0$ , и  $(\theta_j(R + 0, \varphi) = \partial_\rho \theta_j(R + 0, \varphi) = 0)$ , then on the basis of (11), we can write the following:

$$\chi_j^{2,-}(\varphi) = \langle \partial_\rho \theta_j(R, \varphi) \rangle = -\kappa^{-1} (f_j(\varphi) - \theta_j(R - 0, \varphi)), \kappa = \lambda/\alpha \quad (12)$$

As a solution to the boundary value problem (6), (12), we will use the discontinuous solution (10) with the jumps obtained here:

$$\theta_j(\rho, \varphi) = \frac{R}{2\pi} \left\{ \int_{-\pi}^{\pi} \frac{f_j(\psi) - \theta_j(R - 0, \psi)}{\kappa} K_0(r_* \mu) d\psi - \partial_\rho \int_{-\pi}^{\pi} \chi_j^{1,-}(\psi) K_0(r_* \mu) d\psi \right\} + F_j(\rho, \varphi), \quad (13)$$

To get an equation to determine an unknown boundary value:

$\chi_j^{1,-}(\psi) = \theta_j(R - 0, \psi)$ , make the limit transition in (13)  $\rho \rightarrow R - 0$ , considering that:

$$\lim_{\rho \rightarrow R} K_0(\mu \sqrt{\rho^2 + R^2 - 2\rho R \cos(\varphi - \psi)}) = K_0(\mu 2^{1/2} R \sqrt{1 - \cos(\varphi - \psi)})$$

$$= K_0(\mu 2R |\sin((\varphi - \psi)/2)|)$$

And that  $\sin((\varphi - \psi)/2) \approx (\varphi - \psi)/2$ , at  $\varphi \rightarrow \psi$ , as well as representing the MacDonald function as:

$$K_0(z) = \ln(2/z) + \ln(2/z) + \sum_{m=0}^{\infty} \left(\frac{z}{2}\right)^m \frac{1}{\Gamma(m+1)m!} + \sum_{m=0}^{\infty} \left(\frac{z}{2}\right)^{2m} \frac{\psi(m+1)}{(m!)^2}$$

We can say that:

$$\left[ \partial_{\rho} \int_{-\pi}^{\pi} \chi_j^{1,-}(\psi) K_0(r_* \mu) d\psi \right]_{\rho=R \pm 0} = \pm \frac{v \chi_j^{1,-}(\varphi)}{2}; v = -1 - \gamma/2$$

where  $\gamma$  – Euler constant.

As a result, we obtain:

$$\theta_j(\rho, \varphi) = -\frac{R}{2\pi} \left\{ \int_{-\pi}^{\pi} \frac{f_j(\psi) - \chi_j^{1,-}(\psi)}{\kappa} K_0(\mu |\sin((\varphi - \psi)/2)|) d\psi + \frac{v \chi_j^{1,-}(\varphi)}{2} \right\} + F_j(R, \varphi),$$

This relation can be written as an integral equation:

$$\chi_j^{1,-}(\varphi) + \frac{R}{2\pi} \int_{-\pi}^{\pi} \frac{f_j(\psi) - \chi_j^{1,-}(\psi)}{\kappa(v-2)} K_0(\mu |\sin((\varphi - \psi)/2)|) d\psi = F_j(R, \varphi),$$

Using the MacDonald function representation, in the form:

$$K_0(\mu |\sin((\varphi - \psi)/2)|) = \int_0^{\infty} \frac{\alpha}{\alpha^2 + \mu^2} \sum_{n=-\infty}^{\infty} J_n(\alpha R) J_n(\alpha R) e^{in(\varphi - \psi)} d\alpha,$$

we apply the finite Fourier transform to this equation, taking into account its properties and the convolution theorem:

$$\begin{aligned} \Phi_n[\partial_{\varphi}^k f] &= (in)^k \Phi_n[f], \Phi_n[f] = \int_{-\pi}^{\pi} f(\zeta) h(\varphi - \zeta) d\zeta = 2\pi f_n h_n, \Phi_n[e^{im\varphi}] \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i\varphi(m-n)} d\varphi = \delta_{m,n} \end{aligned}$$

We obtain:

$$\begin{aligned} \chi_{j,n}^{1,-} &= \tau_n^{-1} [F_{j,n} - f_{j,n}], \tau_n = 1 - \frac{R}{\kappa(v-2)} h_n \\ h_n &= \int_0^{\infty} \frac{\alpha}{\alpha^2 + \mu^2} \sum_{m=-\infty}^{\infty} J_m(\alpha R) J_m(\alpha R) \int_{-\pi}^{\pi} e^{i(m-n)\phi} d\phi = \int_0^{\infty} \frac{\alpha [J_m(\alpha R)]^2}{\alpha^2 + \mu^2} = \\ f_{j,n} &= \int_{-\pi}^{\pi} f_j(\psi) e^{-in\varphi} d\varphi, F_{j,n} = \int_{-\pi}^{\pi} F_j(R, \varphi) e^{-in\varphi} d\varphi \end{aligned}$$

Inverting the Fourier transform, we obtain

$$\chi_{j,n}^{1,-}(\varphi) = \theta_j(R - 0, \varphi) = \int_{-\pi}^{\pi} (F_j(\eta) - f_j(\eta)) \sum_{n=-\infty}^{\infty} \frac{e^{-in(\varphi - \eta)}}{\tau_n} d\eta \quad (14)$$

We introduce the unknown function  $\psi(\varphi)$ , then the boundary conditions (7) are written as follows:

$$\begin{aligned} \theta(+R, \varphi, t) + \kappa \theta'(+R, \varphi, t) &= g_-(\varphi) + \psi_+(\varphi), \\ g_-(\varphi) &= 0, |\varphi| \in (\varphi_1, \varphi_2), g_-(\varphi) = g(\varphi), |\varphi| \notin (\varphi_1, \varphi_2) \\ \psi_+(\varphi) &= \psi(\varphi), |\varphi| \in (\varphi_1, \varphi_2), \psi_+(\varphi) = 0, |\varphi| \notin (\varphi_1, \varphi_2) \end{aligned} \quad (15)$$

If we assume that the functions  $g_-(\varphi)$  and  $\psi(\varphi)$  are known, then we pass to the third main boundary-value problem, the solution of which was obtained above. Using formula (14), we can write the following.

$$\theta_j(R-0, \varphi) = - \int_{-\pi}^{\pi} (g_-(\eta) + \psi_+(\eta)) \Psi(\varphi - \eta) d\eta + \int_{-\pi}^{\pi} F_j(\eta) \Psi(\varphi - \eta) d\eta,$$

$$\Psi(\varphi - \eta) = \sum_{n=-\infty}^{\infty} \frac{e^{-in(\varphi-\eta)}}{\tau_n} d\eta$$

Now satisfying the first boundary condition from (7) and taking into account (15) we obtain the integral equation:

$$\frac{\partial}{\partial \varphi} \int_{\varphi_1}^{\varphi_2} \psi_j(\eta) \Psi(\varphi - \eta) d\eta = F(\varphi),$$

$$F(\varphi) = \int_{-\pi}^{\pi} F_j(\eta) \Psi(\varphi - \eta) d\eta - \frac{\partial}{\partial \varphi} \left( \int_{-\pi}^{\varphi_1} + \int_{\varphi_2}^{\pi} \right) g(\eta) \Psi(\varphi - \eta) d\eta$$

In fig. Figures 2–4 show plots of temperature changes depending on the polar angle  $\varphi$  and radius  $\rho$  and various values of the temperature of the boundary layer Tbl; moreover, Figures 2 show plots at an angular velocity of  $\omega = 50$  m/c, and in Fig. 3 at  $\omega = 80$  m/c. The temperature of the boundary layer was varied using the Ranque-Hills tube. The temperature was determined at an operating time of 60 s.

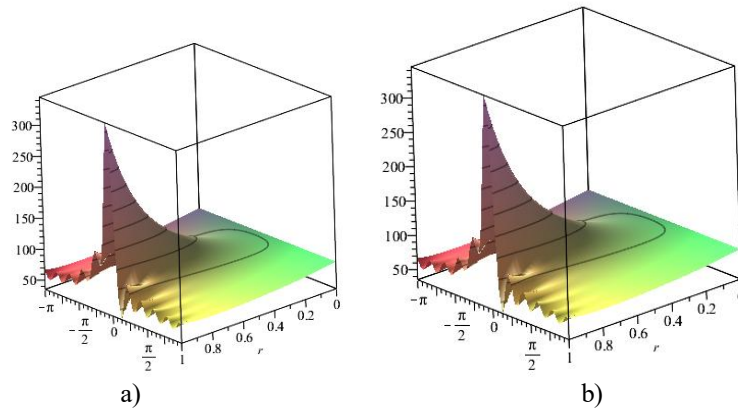


Figure 2.a) The temperature of the circle at the temperature of the boundary layer + 200 °C; b) the temperature of the circle at the temperature of the boundary layer –500 °C. V circle = 50 m/s.



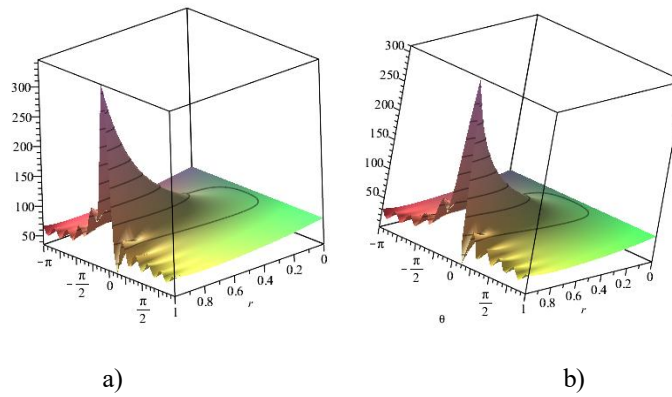


Figure 3 a) The temperature of the circle at the temperature of the boundary layer + 20° C; b) the temperature of the circle at the temperature of the boundary layer -50° C.  $V_{\text{wheel}} = 80 \text{ m / s}$

Additional mathematical modeling of the process of introducing the boundary layer of atomized coolant (fog) showed that the temperature of the disk decreases significantly, as shown in Fig. 4.

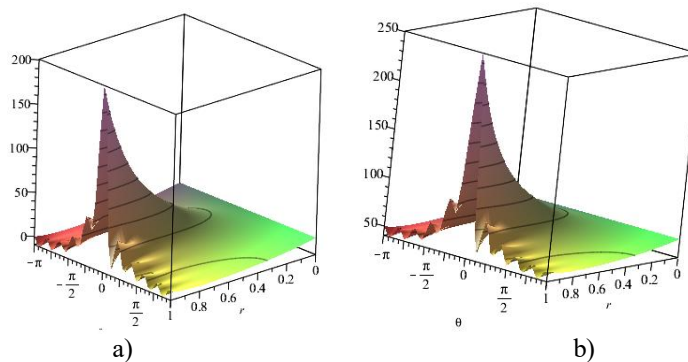


Figure 4 a) Circle temperature when sprayed coolant is introduced into the boundary layer (fog)  $V_{\text{wheel}} = 50 \text{ m / s}$ ; Circle temperature when sprayed coolant (fog) is introduced into the boundary layer.  $V_{\text{wheel}} = 80 \text{ m / s}$ .

## 2 Conclusions.

Contact heat transfer between the circle and the boundary layer is not effective enough to reduce the temperature of the disk

When air with a negative temperature is introduced into the boundary layer by means of a Ranque-Hillsch tube, the disk temperature decreases by about 10%.

A slight decrease in temperature during contact heat transfer between the cutting disc and the boundary layer is explained by a low coefficient of thermal conductivity of air. When a sprayed coolant is introduced into the boundary layer using an ejector tube (fog cooling), the disk temperature decreases by 25%, which ensures an increase in the time of continuous operation.

### 3 Results

As a result of the study, it was found that to increase the time of continuous operation of the disk, cooling of the boundary layer must be carried out using an ejector tube.

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