An Adaptive δ-GLMB Filter Under Noise Statistics Mismatch

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***Abstract*—Aiming at the problem of multi-target tracking under noise statistics mismatch, an adaptive δ-GLMB filter based on variational Bayesian (VB) approach is proposed. The joint distribution of predicted state and corresponding predicted error covariance matrix, and the joint distribution of measurement noise mean vector and covariance matrix are modeled as the Normal-inverse Wishart (NIW) distributions, in which the latent variables are described as the Gamma distributions. In this paper, the single-target filtering density is expressed as the mixture of Normal inverse Wishart inverse Wishart Gamma Gamma (NNIWNIWGG), and an NNIWNIWGG mixture implementation of δ-generalized labeled multi-Bernoulli (δ-GLMB) filter for linear Gaussian is given. According to the minimization of Kullback-Leibler divergence, the approximate solution of predicted likelihood is obtained. Simulation results show that the proposed adaptive δ-GLMB filter has high tracking accuracy in the case of noises statistics mismatch.**

***Index Terms*—variational Bayesian, δ-GLMB filter, Normal-inverse Wishart, noise statistics mismatch, multi-target tracking**

I. Introduction

In the multi-target Bayesian filter based on random finite set (RFS) method, it is normally assumed that the noise statistics are exactly known, such as mean vectors and covariance matrices. However, in the practical multi-target tracking (MTT), the accurate prior knowledge of the noise statistics is not exactly known or completely unknown due to the external unknown disturbances caused by target maneuvers and disturbance-corrupted measurements. [1].

The variational Bayesian (VB) approach is not only used to obtain the approximate posterior density for which the analytical solution does not exist, but also is employed in the situations, where the measurement noise statistics are unknown [2], [3], [4], [5]. Therefore, the VB method is commonly introduced into the random finite set (RFS) framework to deal with the MTT problems with unknown noise statistics. The adaptive probability hypothesis density (PHD) filters based on VB approach were proposed in [6], [7], [8]. Aiming at the problem of MTT in which the measurement noise covariance matrix is an unknown diagonal matrix, Li *et al.* [9] and Wu *et al.* [10] modeled it as an inverse Gamma distribution, and a Gaussian inverse Gamma mixture PHD filter is given by VB method. Yang *et al.* [11] modeled the single-target posterior density as a mixture of Gaussian and inverse Gamma distribution based on VB approximation and cardinality balanced multitarget multi-Bernoulli (CBMeMBer) filter, and proposed an improved CBMeMBer filter which is suitable for the diagonal and unknown measurement noise covariance matrix. Ardeshiri *et al.* [12] presented an adaptive PHD filter suitable for non-diagonal matrix, in which the measurement noise covariance matrix is modeled by inverse Wishart distribution. To distinguish the multi-target trajectories, Qiu *et al.* [13] introduced the VB approximation method into the labeled multi-Bernoulli (LMB) filtering framework, and the Gaussian-inverse Gamma implementation of LMB filter is given. Yuan *et al.* [14] modeled the measurement noise covariance matrix as the inverse Wishart distribution, and proposed the Gaussian inverse Wishart mixture implementation of the δ-GLMB filter for linear model by the VB method.

The MTT filter mentioned above only considered the situation that the measurement noise covariance matrix is unknown. In order to solve the problem of MTT with unknown process and measurement noises covariance matrices, Zhang *et al.* [15] proposed a robust δ-GLMB filter under mismatches in both dynamic and measurement models which drew lessons from the idea of Huang *et al.* [16] that in order to avoid directly estimating the process noise covariance matrix, estimating the covariance matrix of one-step predicted state error. Similarly, Li *et al.* [17] proposed a robust Poisson multi-Bernoulli mixture filter for MTT in the case of inaccurate process and measurement noises covariance matrices based on VB approach by modeling the joint multi-target filtering density as the Gaussian-inverse Wishart-inverse Wishart distribution [18], [19].

Although the existing MTT filters indicate satisfactory tracking performance when the process and measurement noises covariance matrices are inaccurate, they do not consider the inaccuracy and time-varying of the noises mean vectors. For this reason, the problem of MTT under noises statistics mismatch in the framework of δ-GLMB filtering is proposed by expressing the single-target filtering density as the mixture of Gaussian (Normal) inverse Wishart inverse Wishart Gamma Gamma (NNIWNIWGG) via the VB approximation. According to the minimization of Kullback-Leibler divergence, the approximate solution of the predicted likelihood is obtained, and then the NNIWNIWGG implementation of the proposed adaptive δ-GLMB filter is given. Finally, the effectiveness of the proposed adaptive δ-GLMB filter is verified in the simulation scene of noises statistics mismatch.

The rest of this paper is organized as follows. Section II presents a brief description of the background. Section III presents an adaptive δ-GLMB filter based on VB method. The implementation of the proposed adaptive δ-GLMB filter is given in Section IV. Section V verifies the effectiveness of the proposed adaptive δ-GLMB filter by numerical simulation. Finally, the conclusions are given in Section VI.

II. Background

A. Problem Formulation

Considering a linear state-space system given by

$$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1} \tag{1}$$

$$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{x}_k \in \mathbb{R}^{n_{\boldsymbol{x}}}$ is the state vector, $\boldsymbol{F}_{k-1}$ denotes the state transition matrix, $\boldsymbol{w}_{k-1} \sim \mathcal{N}(\boldsymbol{w}_{k-1}; \boldsymbol{q}_{k-1}, \boldsymbol{Q}_{k-1})$ is the Gaussian process noise with mean vector $\boldsymbol{q}_{k-1}$ and covariance matrix $\boldsymbol{Q}_{k-1}$; $\boldsymbol{z}_k \in \mathbb{R}^{n_{\boldsymbol{z}}}$ is the measurement vector, $\boldsymbol{H}_k$ is the measurement matrix, and $\boldsymbol{v}_k \sim \mathcal{N}(\boldsymbol{v}_k; \boldsymbol{r}_k, \boldsymbol{R}_k)$ is the Gaussian measurement noise, and its mean vector and covariance matrix are $\boldsymbol{r}_k$ and $\boldsymbol{R}_k$, respectively. In addition, it is assumed that the initial state vector $\boldsymbol{x}_0 \sim \mathcal{N}(\boldsymbol{x}_k; \boldsymbol{x}_0, \boldsymbol{P}_0)$, $\boldsymbol{w}_k$, and $\boldsymbol{v}_k$ are independent of each other.

For convenience, the dynamic model shown in (1) and the measurement model shown in (2) can be described as the form of probability density function (PDF) given by

$$p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) = \mathcal{N}(\boldsymbol{x}_k; \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{q}_{k-1}, \boldsymbol{Q}_{k-1}) \tag{3}$$

$$p(\boldsymbol{z}_k|\boldsymbol{x}_k) = \mathcal{N}(\boldsymbol{z}_k; \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{r}_k, \boldsymbol{R}_k) \tag{4}$$

where $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ and $p(\boldsymbol{z}_k|\boldsymbol{x}_k)$ indicate the state transition PDF and measurement likelihood PDF respectively.

For target tracking, it is most often assumed that the noise Statistics are accurately known. However, due to the strong maneuver of the target, external interference or sensor failures and so on [20], the true values of the noise mean vectors and the corresponding covariance matrix are generally unknown or not accurately known, or time-varying in practical applications. In practice, the nominal values of the mean vector and its covariance matrix of process noise and measurement noise are generally given according to engineering experience. When the nominal value is close to the real value, a better tracking effect can be obtained, and vice versa.

TABLE I
The Algorithm of Kalman Filter

Steps	Equations
Predicted state estimate	$\hat{\boldsymbol{x}}_{k\|k-1} = \boldsymbol{F}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{q}_{k-1}$
Predicted error covariance	$\boldsymbol{P}_{k\|k-1} = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}\boldsymbol{F}_{k-1}^{\mathrm{T}} + \boldsymbol{Q}_{k-1}$
Kalman gain	$\boldsymbol{K}_k = \boldsymbol{P}_{k\|k-1}\boldsymbol{H}_k^{\mathrm{T}}(\boldsymbol{H}_k\boldsymbol{P}_{k\|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_k)^{-1}$
Updated state estimate	$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k\|k-1} + \boldsymbol{K}_k(\boldsymbol{z}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_{k\|k-1} - \boldsymbol{r}_k)$
Updated error covariance	$\boldsymbol{P}_k = \boldsymbol{P}_{k\|k-1} - \boldsymbol{K}_k\boldsymbol{H}_k\boldsymbol{P}_{k\|k-1}$

For the linear system model shown in (1) and (2), $\boldsymbol{x}_k$ can be estimated according to the Table I for all measurements up to time k represented by $\boldsymbol{z}_{1:k} = \boldsymbol{z}_1, \ldots, \boldsymbol{z}_k$. It can be seen from Table I that the process noise mean vector $\boldsymbol{q}_{k-1}$ indirectly affects the updated state $\hat{\boldsymbol{x}}_k$ through the predicted state $\hat{\boldsymbol{x}}_{k|k-1}$, while the measurement noise mean vector $\boldsymbol{r}_k$ directly impacts $\hat{\boldsymbol{x}}_k$. Moreover, the process noise covariance matrix $\boldsymbol{Q}_{k-1}$ also indirectly affects $\hat{\boldsymbol{x}}_k$ by the predicted error covariance matrix $\boldsymbol{P}_{k|k-1}$, and the measurement noise covariance matrix $\boldsymbol{R}_k$ has a direct effect on $\hat{\boldsymbol{x}}_k$ via the Kalman gain $\boldsymbol{K}_k$.

Due to the uncertainty of process noise, the one-step prediction PDF can be written as

$$p(\boldsymbol{x}_k|\hat{\boldsymbol{x}}_{k|k-1}, \boldsymbol{P}_{k|k-1}, \boldsymbol{z}_{1:k-1}) = \mathcal{N}(\boldsymbol{x}_k; \hat{\boldsymbol{x}}_{k|k-1}, \boldsymbol{P}_{k|k-1}) \tag{5}$$

where the mean vector $\hat{\boldsymbol{x}}_{k|k-1}$ and the prediction error covariance matrix $\boldsymbol{P}_{k|k-1}$ can be calculated by Table 1.

To estimate the target state $\boldsymbol{x}_k$, the analytical solution of the joint posterior density $p(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_{k|k-1}, \boldsymbol{P}_{k|k-1}, \boldsymbol{r}_k, \boldsymbol{R}_k|\boldsymbol{z}_{1:k})$ needs to be obtained. However, due to the coupling relationship between $\boldsymbol{x}_k$ and the unknown parameters such as $\hat{\boldsymbol{x}}_{k|k-1}$, $\boldsymbol{P}_{k|k-1}$, $\boldsymbol{r}_k$ and $\boldsymbol{R}_k$, it is generally difficult to obtain a closed-formed solution. The VB method can use probability graph model [21] and mean field theory [22] to approximate the real a posteriori density by variational approximation density of unknown parameters. Therefore, VB approximation is introduced into RFS framework to deal with the coupling between state $\boldsymbol{x}_k$ and unknown parameters or latent variables.

B. Bayesian Hierarchical Model

In order to ensure that the prior and posterior distribution have the same form, the exponential cluster conjugate prior distribution is used as the joint prior distribution of $\{\hat{\boldsymbol{x}}_{k|k-1}, \boldsymbol{P}_{k|k-1}\}$ and $\{\boldsymbol{r}_k, \boldsymbol{R}_k\}$ under the conjugate condition. In the sense of Bayesian statistics, Gaussian-inverse Wishart (NIW) distribution is a conjugate prior of multidimensional Gaussian distribution with unknown mean and covariance matrix, so the NIW distribution is chosen as the joint prior distribution of $\hat{\boldsymbol{x}}_{k|k-1}$ and $\boldsymbol{P}_{k|k-1}$, which can be written as

$$\begin{aligned} p(\hat{\boldsymbol{x}}_{k|k-1}, \boldsymbol{P}_{k|k-1}|\boldsymbol{z}_{1:k-1}) =& \mathcal{N}(\hat{\boldsymbol{x}}_{k|k-1}; \boldsymbol{s}_k, \boldsymbol{P}_{k|k-1}/\alpha_k) \\ &\times \mathcal{IW}(\boldsymbol{P}_{k|k-1}; t_k, \boldsymbol{T}_k) \end{aligned} \tag{6}$$

in which $\mathcal{IW}(\cdot; \sigma, \boldsymbol{\Sigma})$ means the inverse Wishart distribution with the degree of freedom parameter σ and the inverse scale matrix $\boldsymbol{\Sigma}$. The latent variable α_k represents the prior confidence parameter, and the Gamma distribution is selected as its prior distribution as follow

$$p(\alpha_k|\boldsymbol{z}_{1:k-1}) = \mathcal{G}(\alpha_k; a_k, b_k) \tag{7}$$

where $\mathcal{G}(\cdot|a, b)$ represents the Gamma distribution with shape parameter a and ratio parameter b. Similarly, the joint prior density of $\boldsymbol{r}_k$and $\boldsymbol{R}_k$ is modeled as the Gaussian-inverse Wishart form

$$p(\boldsymbol{r}_k, \boldsymbol{R}_k|\boldsymbol{z}_{1:k-1}) = \mathcal{N}(\boldsymbol{r}_k; \boldsymbol{\varepsilon}_k, \boldsymbol{R}_k/\beta_k)\mathcal{IW}(\boldsymbol{R}_k; u_k, \boldsymbol{U}_k) \tag{8}$$

where the latent variable β_k also represents the prior confidence parameter, and the Gamma distribution is also selected as its prior PDF given by

$$p(\beta_k|\boldsymbol{z}_{1:k-1}) = \mathcal{G}(\beta_k; c_k, d_k) \tag{9}$$

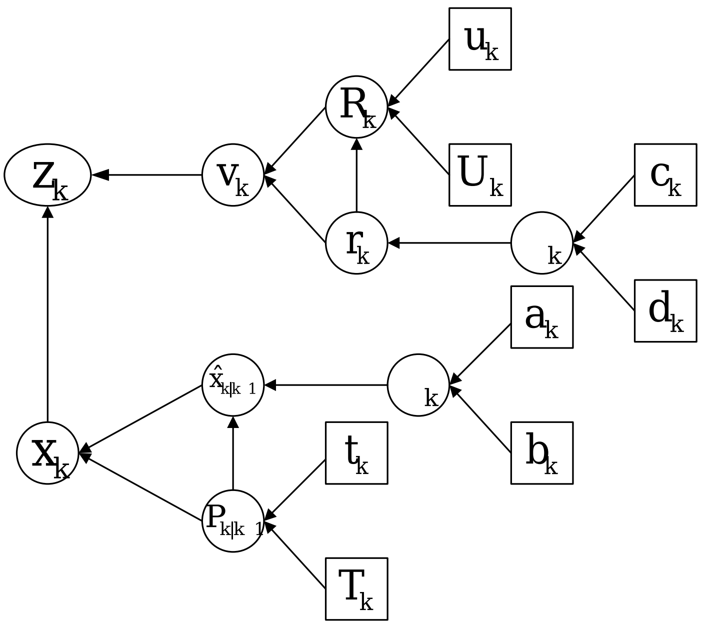

Fig. 1. Hierarchical Bayesian model under noise statistics mismatch

According to (4)–(9), the hierarchical Bayesian model of single target tracking at time k under noise statistics mismatch is shown in Fig.1, in which the ellipse means the measurement vector, the circle represents the parameters to be estimated, and the box denotes the given hyper-parameter. As a structured statistical model, Bayesian hierarchical model is commonly utilized to solve complex statistical problems.

III. δ-GLMB filtering Under Noise Statistics Mismatch

A. NNIWNIWGG Model

In order to estimate the state $\boldsymbol{x}_k$ and unknown parameters simultaneously, they must be expressed as augmented state on the mixed state space, and the corresponding joint a posterior density function can be expressed in the form of Gaussian inverse Wishart inverse Wishart Gamma Gamma (NNIWNIWGG) distribution. Using a state modeling method similar to that in extended target tracking, the hybrid state space is constructed as follow

$$\begin{aligned}\boldsymbol{\zeta}_k \triangleq &\{\boldsymbol{x}_k, \hat{\boldsymbol{x}}_{k|k-1}, \boldsymbol{P}_{k|k-1}, \boldsymbol{r}_k, \boldsymbol{R}_k, \alpha_k, \beta_k\} \\ &\in \mathbb{R}^{n_{\boldsymbol{x}}} \times \mathbb{R}^{n_{\boldsymbol{x}}} \times \mathbb{R}_+^{n_{\boldsymbol{x}}} \times \mathbb{R}^{n_{\boldsymbol{z}}} \times \mathbb{R}_+^{n_{\boldsymbol{z}}} \times \mathbb{R}^+ \times \mathbb{R}^+\end{aligned} \tag{10}$$

where $\boldsymbol{\zeta}_k$ represents the augmented state$\mathbb{R}^{n_{\boldsymbol{x}}}$ represents the dimensional real vector space of $n_{\boldsymbol{x}}$, $\mathbb{R}_+^{n_{\boldsymbol{x}}}$ represents the positive real number space of $n_{\boldsymbol{x}} \times n_{\boldsymbol{x}}$, $\mathbb{R}^+$ represents the positive real number space. For the single target tracking problem, the joint posterior density of the augmented state at time k can be expressed as the NNIWNIWGG mixed distribution

$$\begin{aligned}p(\boldsymbol{\zeta}_k) = &\mathcal{N}(\boldsymbol{x}_k; \boldsymbol{m}_k, \boldsymbol{\Sigma}_k)\mathcal{N}(\hat{\boldsymbol{x}}_{k|k-1}; \boldsymbol{s}_k, \boldsymbol{S}_k) \\ &\times \mathcal{IW}(\boldsymbol{P}_{k|k-1}; t_k, \boldsymbol{T}_k)\mathcal{N}(\boldsymbol{r}_k; \boldsymbol{\lambda}_k, \boldsymbol{\Lambda}_k) \\ &\times \mathcal{IW}(\boldsymbol{R}_k; u_k, \boldsymbol{U}_k)\mathcal{G}(\alpha_k; a_k, b_k)\mathcal{G}(\beta_k; c_k, d_k) \\ \triangleq &\mathcal{NNIWNIWGG}(\boldsymbol{\zeta}_k; \boldsymbol{\phi}_k)\end{aligned} \tag{11}$$

where $\boldsymbol{\phi}_k = (\boldsymbol{m}_k, \boldsymbol{\Sigma}_k, \boldsymbol{s}_k, \boldsymbol{S}_k, t_k, \boldsymbol{T}_k, \boldsymbol{\lambda}_k, \boldsymbol{\Lambda}_k, u_k, \boldsymbol{U}_k, a_k, b_k, c_k, d_k)$ denotes the hyper-parameter array which encapsulates all the statistics of the NNIWNIWGG model.

B. The proposed Adaptive δ-GLMB Filter

Different from the standard δ-GLMB filter, a joint estimation of the target state and other parameters is indispensable to the adaptive δ-GLMB filter with uncertain noise statistical characteristics . According to the view of multi-target Bayesian filtering, it is necessary to transfer the multi-target filtering density in the form of δ-GLMB RFS forward over time.The labeled multi-target states and parameters can be expressed as $\boldsymbol{\Phi} = \{(\boldsymbol{\zeta}, \ell)_j\}(j = 1, 2, \ldots, N)$,where N is the number of target states. Multi-target measurements can be modeled as RFS, as $Z = \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{|Z|}\}$, where $\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{|Z|}$ means every single measurement. When the statistical characteristics of noise are uncertain, the prediction and update of adaptive δ-GLMB filter are given by proposition 1 and proposition 2 respectively.

In order to express succinctly, the time subscript is omitted and the one-step prediction of the variable is represented by '+'.

Proposition 1: If the joint posterior density function at the previous time of multi-object has the δ-GLMB form as follow:

$$\pi(\boldsymbol{\Phi}) = \Delta(\boldsymbol{\Phi}) \sum_{(I,\vartheta)\in\mathcal{F}(\mathbb{L})\times\Xi} w^{(I,\vartheta)}\delta_I(\mathcal{L}(\boldsymbol{\Phi}))[p^{(\vartheta)}]^{\boldsymbol{\Phi}} \tag{12}$$

then the augmented state joint predictive density function at this time also has the same form

$$\pi_+(\boldsymbol{\Phi}_+) = \Delta(\boldsymbol{\Phi}_+) \sum_{(I_+,\vartheta)\in\mathcal{F}(\mathbb{L}_+)\times\Xi} w_+^{(I_+,\vartheta)}\delta_{I_+}(\mathcal{L}(\boldsymbol{\Phi}_+))[p_+^{(\vartheta)}]^{\boldsymbol{\Phi}_+} \tag{13}$$

with

$$w_+^{(I_+,\vartheta)} = w_{\mathrm{S}}^{(\vartheta)}(I_+ \bigcap \mathbb{L})w_\Gamma(I_+ \bigcap \mathbb{B}) \tag{14}$$

$$w_{\mathrm{S}}^{(\vartheta)}(J) = [\eta_{\mathrm{S}}^{(\vartheta)}]^J \sum_{I\supseteq J} [1-\eta_{\mathrm{S}}^{(\vartheta)}]^{I-J} w^{(I,\vartheta)} \tag{15}$$

$$p_+^{(\vartheta)}(\boldsymbol{\zeta}, \ell) = \mathbf{1}_{\mathbb{L}}(\ell)p_{+,\mathrm{S}}^{(\vartheta)}(\boldsymbol{\zeta}, \ell) + (1-\mathbf{1}_{\mathbb{L}}(\ell))p_\Gamma(\boldsymbol{\zeta}, \ell) \tag{16}$$

$$p_{+,\mathrm{S}}^{(\vartheta)}(\boldsymbol{\zeta}, \ell) = \frac{\left\langle P_{\mathrm{S}}(\cdot, \ell)f(\boldsymbol{\zeta}|\cdot, \ell), p^{(\vartheta)}(\cdot, \ell)\right\rangle}{\eta_{\mathrm{S}}^{(\vartheta)}(\ell)} \tag{17}$$

$$\eta_{\mathrm{S}}^{(\vartheta)}(\ell) = \int \left\langle P_{\mathrm{S}}(\cdot, \ell)f(\boldsymbol{\zeta}|\cdot, \ell), p^{(\vartheta)}(\cdot, \ell)\right\rangle d\boldsymbol{\zeta} \tag{18}$$

where $w_+^{(I_+,\vartheta)}$ denotes the weight of the predictive hypothetical component (I_+, ϑ) , $w_{\mathrm{S}}^{(\vartheta)}(J)$ represents the weight of the surviving labeled collection J , $w_\Gamma(\cdot)$ represents the weight of the new born labeled collection, $\eta_{\mathrm{S}}^{(\vartheta)}(\cdot)$ denotes normalized constant, $p_{+}^{(\vartheta)}(\boldsymbol{\zeta},\ell)$ denotes the augmented state joint predicted density function of single target, $P_{\mathrm{S}}(\cdot,\ell)$ denotes the survival probability of the target, $p_{+,\mathrm{S}}^{(\vartheta)}(\boldsymbol{\zeta},\ell)$ denotes the joint predicted density function of surviving targets , $p_{\Gamma}(\cdot,\ell)$ denotes the probability density function of new born targets, $f(\boldsymbol{\zeta}|\cdot,\ell)$ denotes the augmented state transfer density of single target.

Proposition 2: If the joint multi-target predicted density also has the δ-GLMB form as shown in (12), then the joint multi-target posterior density is given by

$$\pi(\boldsymbol{\Phi})=\Delta(\boldsymbol{\Phi})\sum_{(I,\vartheta)\in\mathcal{F}(\mathbb{L})\times\Xi}\sum_{\theta\in\Theta(I)}w^{(I,\vartheta,\theta)}(Z)\delta_{I}(\mathcal{L}(\boldsymbol{\Phi}))[p^{(\vartheta,\theta)}(\cdot|Z)]^{\boldsymbol{\Phi}} \tag{19}$$

with

$$w^{(I,\vartheta,\theta)}(Z)=\frac{\delta_{\theta^{-1}(\{0:|Z|\})}(I)[\eta_{Z}^{(\vartheta,\theta)}]^{I}w^{(I,\vartheta)}}{\sum\limits_{(I,\vartheta)\in\mathcal{F}(\mathbb{L})\times\Xi}\sum\limits_{\theta\in\Theta(I)}\delta_{\theta^{-1}(\{0:|Z|\})}(I)[\eta_{Z}^{(\vartheta,\theta)}]^{I}w^{(I,\vartheta)}} \tag{20}$$

$$\eta_{Z}^{(\vartheta,\theta)}(\ell)=\left\langle p_{+}^{(\vartheta)}(\boldsymbol{\zeta},\ell),\varphi_{Z}(\boldsymbol{\zeta},\ell;\theta)\right\rangle \tag{21}$$

$$p^{(\vartheta,\theta)}(\boldsymbol{\zeta},\ell|Z)=\frac{p_{+}^{(\vartheta)}(\boldsymbol{\zeta},\ell)\varphi_{Z}(\boldsymbol{\zeta},\ell;\theta)}{\eta_{Z}^{(\vartheta,\theta)}(\ell)} \tag{22}$$

$$\varphi_{Z}(\boldsymbol{\zeta},\ell;\theta)=\begin{cases}\frac{P_{\mathrm{D}}(\boldsymbol{\zeta},\ell)p(\boldsymbol{z}_{\theta(\ell)}|\boldsymbol{\zeta},\ell)}{\boldsymbol{\kappa}(\boldsymbol{z}_{\theta(\ell)})}, & \theta(\ell)>0\\ 1-P_{\mathrm{D}}(\boldsymbol{\zeta},\ell), & \theta(\ell)=0\end{cases} \tag{23}$$

where Z represents the set of measurements received by the sensor at the current time, $w^{(I,\vartheta,\theta)}(Z)$ denotes the update weight of the hypothetical component(I,ϑ,θ) , $\eta_{Z}^{(\vartheta,\theta)}$ denotes normalized constant, $p^{(\vartheta,\theta)}(\cdot,\ell|Z)$ denotes the augmented state joint posterior density function of single target, $P_{\mathrm{D}}(\cdot,\ell)$ indicates the detection probability of the sensor, $\boldsymbol{\kappa}(\cdot)$ represents the clutter intensity function, $p(\cdot|\boldsymbol{\zeta},\ell)$ denotes the likelihood function of single target measurement, $\varphi_{Z}(\cdot,\ell;\theta)$ denotes the generalized measurement likelihood function.

IV. IMPLEMENTATION

To obtain the implementation of the proposed adaptive δ-GLMB filter, the following assumptions are given:

Assumption 1: The single-target transition density $f(\boldsymbol{\zeta}_k|\boldsymbol{\zeta}_{k-1})$ is Markovian, which can be written as

$$\begin{aligned}f(\boldsymbol{\zeta}_k|\boldsymbol{\zeta}_{k-1})=&f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})f(\hat{\boldsymbol{x}}_{k|k-1}|\boldsymbol{P}_{k|k-1},\alpha_k)\\ &\times f(\boldsymbol{P}_{k|k-1}|\boldsymbol{P}_{k-1|k-2})f(\boldsymbol{r}_k|\boldsymbol{R}_k,\beta_k)\\ &\times f(\boldsymbol{R}_k|\boldsymbol{R}_{k-1})f(\beta_{k-1}|\beta_k)\end{aligned} \tag{24}$$

Assumption 2: The survival and detection probabilities are independent of target augmented state and label, i.e.

$$P_{\mathrm{S}}(\boldsymbol{\zeta},\ell)=P_{\mathrm{S}} \tag{25}$$

$$P_{\mathrm{D}}(\boldsymbol{\zeta},\ell)=P_{\mathrm{D}} \tag{26}$$

Assumption 3: The multi-target birth model can be expressed as an NNIWNIWGG mixture in the following form

$$p_{\Gamma}(\boldsymbol{\zeta},\ell)=\sum_{j=1}^{J_{\Gamma}(\ell)}w_{\Gamma}^{(j)}(\ell)\mathcal{NNIWNIWGG}(\boldsymbol{\zeta};\boldsymbol{\phi}_{\Gamma}^{(j)}(\ell)) \tag{27}$$

where J_{Γ}, $w_{\Gamma}^{(j)}$ and $\boldsymbol{\phi}_{\Gamma}^{(j)}=(\boldsymbol{m}_{\Gamma}^{(j)},\boldsymbol{\Sigma}_{\Gamma}^{(j)},\boldsymbol{s}_{\Gamma}^{(j)},\boldsymbol{S}_{\Gamma}^{(j)},t_{\Gamma}^{(j)},\boldsymbol{T}_{\Gamma}^{(j)},\boldsymbol{\lambda}_{\Gamma}^{(j)},\boldsymbol{\Lambda}_{\Gamma}^{(j)},u_{\Gamma}^{(j)},\boldsymbol{U}_{\Gamma}^{(j)},a_{\Gamma}^{(j)},b_{\Gamma}^{(j)},c_{\Gamma}^{(j)},d_{\Gamma}^{(j)})$ are the NNIWNIWGG model parameters which control the shape of the density.

Given the above assumptions, Proposition 3 and Proposition 4 give a closed-form solution of the proposed adaptive δ-GLMB filter under noise statistics mismatch.

Proposition 3: Suppose that the joint multi-target prior density is shown in (12), where the single-target density $p^{(\vartheta)}(\boldsymbol{\zeta},\ell)$ is the NNIWNIWGG mixture with the form

$$p^{(\vartheta)}(\boldsymbol{\zeta},\ell)=\sum_{i=1}^{J^{(\vartheta)}(\ell)}w^{(\vartheta,i)}\mathcal{NNIWNIWGG}(\boldsymbol{\zeta};\boldsymbol{\phi}^{(\vartheta,i)}(\ell)) \tag{28}$$

where $\boldsymbol{\phi}^{(\vartheta,i)}=(\boldsymbol{m}^{(\vartheta,i)},\boldsymbol{\Sigma}^{(\vartheta,i)},\boldsymbol{s}^{(\vartheta,i)},\boldsymbol{S}^{(\vartheta,i)},t^{(\vartheta,i)},\boldsymbol{T}^{(\vartheta,i)},\boldsymbol{\lambda}^{(\vartheta,i)},\boldsymbol{\Lambda}^{(\vartheta,i)},u^{(\vartheta,i)},\boldsymbol{U}^{(\vartheta,i)},a^{(\vartheta,i)},b^{(\vartheta,i)},c^{(\vartheta,i)},d^{(\vartheta,i)})$, then the joint multi-target predicted density is shown in (13), while the single-target predicted density $p_{+}^{(\vartheta)}(\boldsymbol{\zeta},\ell)$ is also NNIWNIWGG mixtures of the form

$$\begin{aligned}p_{+}^{(\vartheta)}(\boldsymbol{\zeta},\ell)=&1_{\mathbb{L}}(\ell)\sum_{i=1}^{J^{(\vartheta)}(\ell)}w^{(\vartheta,i)}\mathcal{NNIWNIWGG}(\boldsymbol{\zeta};\boldsymbol{\phi}_{\mathrm{S}}^{(\vartheta,i)})\\ &+(1-1_{\mathbb{L}}(\ell))p_{\Gamma}(\boldsymbol{\zeta},\ell)\end{aligned} \tag{29}$$

where $\boldsymbol{\phi}_{\mathrm{S}}^{(\vartheta,i)}=(\boldsymbol{m}_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{\Sigma}_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{s}_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{S}_{\mathrm{S}}^{(\vartheta,i)},t_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{T}_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{\lambda}_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{\Lambda}_{\mathrm{S}}^{(\vartheta,i)},u_{\mathrm{S}}^{(\vartheta,i)},\boldsymbol{U}_{\mathrm{S}}^{(\vartheta,i)},a_{\mathrm{S}}^{(\vartheta,i)},b_{\mathrm{S}}^{(\vartheta,i)},c_{\mathrm{S}}^{(\vartheta,i)},d_{\mathrm{S}}^{(\vartheta,i)})$ is the hyper-parameter set in the joint predicted density of survival targets, each of which can be calculated by

$$\boldsymbol{m}_{\mathrm{S}}^{(\vartheta,i)}=\boldsymbol{F}\boldsymbol{m}^{(\vartheta,i)}+\boldsymbol{q}^{*} \tag{30}$$

$$\boldsymbol{\Sigma}_{\mathrm{S}}^{(\vartheta,i)}=\boldsymbol{F}\boldsymbol{\Sigma}^{(\vartheta,i)}\boldsymbol{F}^{\mathrm{T}}+\boldsymbol{Q}^{*} \tag{31}$$

$$\boldsymbol{s}_{\mathrm{S}}^{(\vartheta,i)}=\boldsymbol{m}_{\mathrm{S}}^{(\vartheta,i)} \tag{32}$$

$$\boldsymbol{S}_{\mathrm{S}}^{(\vartheta,i)}=\boldsymbol{S}^{(\vartheta,i)} \tag{33}$$

$$t_{\mathrm{S}}^{(\vartheta,i)}=\tau+n_{\boldsymbol{x}}+1 \tag{34}$$

$$\boldsymbol{T}_{\mathrm{S}}^{(\vartheta,i)}=\tau\boldsymbol{\Sigma}_{\mathrm{S}}^{(\vartheta,i)} \tag{35}$$

$$\boldsymbol{\lambda}_{\mathrm{S}}^{(\vartheta,i)}=\boldsymbol{r}^{*} \tag{36}$$

$$\boldsymbol{\Lambda}_{\mathrm{S}}^{(\vartheta,i)}=\boldsymbol{\Lambda}^{(\vartheta,i)} \tag{37}$$

$$u_{\mathrm{S}}^{(\vartheta,i)}=\rho(u^{(\vartheta,i)}-n_{\boldsymbol{z}}-1)+n_{\boldsymbol{z}}+1 \tag{38}$$

$$\boldsymbol{U}_{\mathrm{S}}^{(\vartheta,i)}=\rho\boldsymbol{U}^{(\vartheta,i)} \tag{39}$$

$$a_{\mathrm{S}}^{(\vartheta,i)}=\rho a^{(\vartheta,i)},b_{\mathrm{S}}^{(\vartheta,i)}=\rho b^{(\vartheta,i)} \tag{40}$$

$$c_{\mathrm{S}}^{(\vartheta,i)}=\rho c^{(\vartheta,i)},d_{\mathrm{S}}^{(\vartheta,i)}=\rho d^{(\vartheta,i)} \tag{41}$$

where $\tau \geq 0$ a tuning parameter, and $\rho \in [0, 1)$ is a forgetting factor which describes the dynamic uncertainty of unknown parameters, and the multi-target birth model $p_{\Gamma}(\boldsymbol{\zeta}, \ell)$ is given by (27).

Proposition 4: Suppose that the joint multi-target predicted density is shown in (13), and the single-target predicted density $p_{+}^{(\vartheta)}(\boldsymbol{\zeta}, \ell)$ can be expressed as the NNIWNNIWGG mixture with the form

$$p_{+}^{(\vartheta)}(\boldsymbol{\zeta}, \ell) = \sum_{i=1}^{J_{+}^{(\vartheta)}} w_{+}^{(\vartheta,i)} \mathcal{NNIWNNIWGG}(\boldsymbol{\zeta}; \boldsymbol{\phi}_{+}^{(\vartheta,i)}) \quad (42)$$

where $\boldsymbol{\phi}_{+}^{(\vartheta,i)} = (\boldsymbol{m}_{+}^{(\vartheta,i)}, \boldsymbol{\Sigma}_{+}^{(\vartheta,i)}, \boldsymbol{s}_{+}^{(\vartheta,i)}, \boldsymbol{S}_{+}^{(\vartheta,i)}, t_{+}^{(\vartheta,i)}, \boldsymbol{T}_{+}^{(\vartheta,i)}, \boldsymbol{\lambda}_{+}^{(\vartheta,i)}, \boldsymbol{\Lambda}_{+}^{(\vartheta,i)}, u_{+}^{(\vartheta,i)}, \boldsymbol{U}_{+}^{(\vartheta,i)}, a_{+}^{(\vartheta,i)}, b_{+}^{(\vartheta,i)}, c_{+}^{(\vartheta,i)}, d_{+}^{(\vartheta,i)})$ denotes the hyper-parameter set in the single-target predicted density, then the joint multi-target posterior density is shown in (19), where the single-target posterior density is given by

$$p^{(\vartheta,\theta)}(\boldsymbol{\zeta}|Z) = \sum_{i=1}^{J_{+}^{(\vartheta)}} \frac{w_{Z,i}^{(\vartheta,\theta)}}{\eta_{Z}^{(\vartheta,\theta)}} \mathcal{NNIWNNIWGG}(\boldsymbol{\zeta}; \boldsymbol{\phi}_{Z,i}^{(\vartheta,\theta)}) \quad (43)$$

where $\boldsymbol{\phi}_{Z,i}^{(\vartheta,\theta)} = (\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)}, \boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)}, \boldsymbol{s}_{Z,i}^{(\vartheta,\theta)}, \boldsymbol{S}_{Z,i}^{(\vartheta,\theta)}, t_{Z,i}^{(\vartheta,\theta)}, \boldsymbol{T}_{Z,i}^{(\vartheta,\theta)}, \boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)}, \boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)}, u_{Z,i}^{(\vartheta,\theta)}, U_{Z,i}^{(\vartheta,\theta)}, a_{Z,i}^{(\vartheta,\theta)}, b_{Z,i}^{(\vartheta,\theta)}, c_{Z,i}^{(\vartheta,\theta)}, d_{Z,i}^{(\vartheta,\theta)})$, and the normalization constant can be calculated by

$$\eta_{Z}^{(\vartheta,\theta)} = \sum_{i=1}^{J_{+}^{(\vartheta)}} w_{Z,i}^{(\vartheta,\theta)} \quad (44)$$

For $\theta(\ell) = 0$, which denotes no measurements are associated with track ℓ, we have $w_{Z,i}^{(\vartheta,\theta)} = w_{+}^{(\vartheta,i)}(1 - P_{\mathrm{D}})$, $\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{m}_{+}^{(\vartheta,i)}$, $\boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{\Sigma}_{+}^{(\vartheta,i)}$, $\boldsymbol{s}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{s}_{+}^{(\vartheta,i)}$, $\boldsymbol{S}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{S}_{+}^{(\vartheta,i)}$, $t_{Z,i}^{(\vartheta,\theta)} = t_{+}^{(\vartheta,i)}$, $\boldsymbol{T}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{T}_{+}^{(\vartheta,i)}$, $\boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{\lambda}_{+}^{(\vartheta,i)}$, $\boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{\Lambda}_{+}^{(\vartheta,i)}$, $u_{Z,i}^{(\vartheta,\theta)} = u_{+}^{(\vartheta,i)}$, $\boldsymbol{U}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{U}_{+}^{(\vartheta,i)}$, $a_{Z,i}^{(\vartheta,\theta)} = a_{+}^{(\vartheta,i)}$, $b_{Z,i}^{(\vartheta,\theta)} = b_{+}^{(\vartheta,i)}$, $c_{Z,i}^{(\vartheta,\theta)} = c_{+}^{(\vartheta,i)}$, $d_{Z,i}^{(\vartheta,\theta)} = d_{+}^{(\vartheta,i)}$.

If $\theta(\ell) > 0$, which means there is a measurement is associated to the track ℓ, then

$$w_{Z,i}^{(\vartheta,\theta)} = q_{i}^{(\vartheta)}(\boldsymbol{z}_{\theta(\ell)}) \frac{w_{+}^{(\vartheta,i)} P_{\mathrm{D}}}{\boldsymbol{\kappa}(\boldsymbol{z}_{\theta(\ell)})} \quad (45)$$

while the parameters in the hyper-parameter set $\boldsymbol{\phi}_{Z,i}^{(\vartheta,\theta)}$ can be calculated by the fixed-point iterations as shown in Table II, and the predicted likelihood $q_{i}^{(\vartheta)}(\boldsymbol{z}_{\theta(\ell)})$ can be obtained using the final values of these parameters according to

$$\begin{aligned} q_{i}^{(\vartheta)}(\boldsymbol{z}_{\theta(\ell)}) &= 0.5 t_{+}^{(\vartheta,i)} \ln |\boldsymbol{T}_{+}^{(\vartheta,i)}| - \ln \Gamma\left(0.5 t_{+}^{(\vartheta,i)}\right) \\ &\quad - 0.5 n_{\boldsymbol{z}} \ln 2\pi + 0.5 u_{+}^{(\vartheta,i)} \ln |\boldsymbol{U}_{+}^{(\vartheta,i)}| \\ &\quad - \ln \Gamma\left(0.5 u_{+}^{(\vartheta,i)}\right) + a_{+}^{(\vartheta,i)} \ln b_{+}^{(\vartheta,i)} - \ln \Gamma(a_{+}^{(\vartheta,i)}) \\ &\quad + c_{+}^{(\vartheta,i)} \ln d_{+}^{(\vartheta,i)} - \ln \Gamma(c_{+}^{(\vartheta,i)}) + 0.5 \ln |\boldsymbol{P}| \\ &\quad + n_{\boldsymbol{x}} + 0.5 \ln |\boldsymbol{S}_{Z,i}^{(\vartheta,\theta)}| - 0.5 t_{Z,i}^{(\vartheta,\theta)} \ln |\boldsymbol{T}_{Z,i}^{(\vartheta,\theta)}| \\ &\quad + n_{\boldsymbol{x}} \ln 2 + \ln \Gamma\left(0.5 t_{Z,i}^{(\vartheta,\theta)}\right) + \ln \Gamma(c_{Z,i}^{(\vartheta,\theta)}) \\ &\quad + 0.5 \mathrm{tr}\left(\mathrm{E}\left[\boldsymbol{P}_{+}{}^{-1}\right] \mathrm{E}[\alpha] \boldsymbol{B}\right) + 0.5 \ln |\boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)}| \\ &\quad + 0.5 n_{\boldsymbol{z}} - 0.5 u_{Z,i}^{(\vartheta,\theta)} \ln |\boldsymbol{U}_{Z,i}^{(\vartheta,\theta)}| + n_{\boldsymbol{z}} \ln 2 \\ &\quad + \ln \Gamma\left(0.5 u_{Z,i}^{(\vartheta,\theta)}\right) + 0.5 \mathrm{tr}\left(\mathrm{E}\left[\boldsymbol{R}^{-1}\right] \mathrm{E}[\beta] \boldsymbol{D}\right) \\ &\quad - a_{Z,i}^{(\vartheta,\theta)} \ln b_{Z,i}^{(\vartheta,\theta)} + \ln \Gamma(a_{Z,i}^{(\vartheta,\theta)}) - c_{Z,i}^{(\vartheta,\theta)} \ln d_{Z,i}^{(\vartheta,\theta)} \end{aligned} \quad (46)$$

The predicted likelihood $q_{i}^{(\vartheta)}(\boldsymbol{z}_{\theta(\ell)})$ can be obtained via minimizing the Kullback-Leibler divergence by the variational lower bound, while its derivation is similar to the Appendix A in [9] and the Appendix D in [23].

V. Numerical Simulation

In this section, the performance of the proposed NNIWNNIWGG-δ-GLMB filter is illustrated in a multi-target tracking scenario with unknown and time-varying noise statistics. We compare it with the existing NIWIW-δ-GLMB filter with nominal process and measurement noises mean [15], as well as the GM-δ-GLMB filter with nominal process and measurement noise mean vectors and covariance matrices. Moreover, the GM-δ-GLMB filter with true noise mean vectors and covariance matrices is utilized as the reference standard for multi-target tracking performance. The cardinality statistics and the Optimal Subpattern Assignment (OSPA) distance [24] are selected to evaluate these filters estimation performance with the cut-off parameter $c = 100$ and the order parameter $p = 2$.

Considering a two-dimensional surveillance region [-1000, 1000]m×[-1000, 1000]m, the kinematic state $\boldsymbol{x}_k = [p_{\boldsymbol{x},k}, \dot{p}_{\boldsymbol{x},k}, p_{\boldsymbol{y},k}, \dot{p}_{\boldsymbol{y},k}]^T$ of each target includes the position $[p_{\boldsymbol{x},k}, p_{\boldsymbol{y},k}]$ and velocity $[\dot{p}_{\boldsymbol{x},k}, \dot{p}_{\boldsymbol{y},k}]$. In addition, the sensor is placed in the center of the surveillance region, and there is miss detection and clutter interference. The true trajectories of targets are shown in Fig.2. Different colored lines denote different targets. And the circles and triangles are the starting and ending points of each target, respectively.

The single-target Markov state transition PDF is given by (3) ,where the state transition matrix

$$F_{k-1} = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad (47)$$

TABLE II
ITERATION STEP OF NNIWNIWGG-δ-GLMB UPDATE

Initialization: $\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)^{(0)}} = \boldsymbol{m}_{+}^{(\vartheta,i)}$, $\boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)^{(0)}} = \boldsymbol{\Sigma}_{+}^{(\vartheta,i)}$,
$\bar{\boldsymbol{P}}^{(0)} = \boldsymbol{\Sigma}_{+}^{(\vartheta,i)}$, $\bar{\boldsymbol{R}}^{(0)} = \boldsymbol{U}^{(\vartheta,i)}/(\boldsymbol{u}^{(\vartheta,i)} - n_{\boldsymbol{z}} - 1)$,
$\mathrm{E}^{(0)}[\alpha] = a^{(\vartheta,i)}/b^{(\vartheta,i)}$, $\mathrm{E}^{(0)}[\beta] = c^{(\vartheta,i)}/d^{(\vartheta,i)}$

for j=0:N-1

$\boldsymbol{s}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \frac{\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)^{(j)}} + \boldsymbol{s}_{+}^{(\vartheta,i)}\mathrm{E}^{(j)}[\alpha]}{1+\mathrm{E}^{(i)}[\alpha_k]}$,

$\boldsymbol{S}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \frac{\bar{\boldsymbol{P}}^{(j)}}{1+\mathrm{E}^{(j)}[\alpha]}$,

$\boldsymbol{A}^{(j+1)} = \bar{\boldsymbol{P}}^{(j)} + (\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)^{(j)}} - \boldsymbol{s}_{Z,i}^{(\vartheta,\theta)^{(j+1)}})(\cdot)^{\mathrm{T}}$

$\boldsymbol{B}^{(j+1)} = \boldsymbol{S}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} + (\boldsymbol{s}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} - \boldsymbol{s}_{+}^{(\vartheta,i)})(\cdot)^{\mathrm{T}}$

$\boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \frac{\boldsymbol{\lambda}_{+}^{(\vartheta,i)}\mathrm{E}^{(j)}[\beta] + (\boldsymbol{z} - \boldsymbol{H}\mathrm{E}^{(j)}[\boldsymbol{x}])}{1+\mathrm{E}^{(j)}[\beta]}$,

$\boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \frac{\bar{\boldsymbol{R}}^{(j)}}{1+\mathrm{E}^{(j)}[\beta]}$,

$\boldsymbol{C}^{(j+1)} = (\boldsymbol{z} - \boldsymbol{H}\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} - \boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}})(\cdot)^{\mathrm{T}} + \boldsymbol{H}\boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)^{(j+1)}}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}}$,

$\boldsymbol{D}^{(j+1)} = \boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} + (\boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} - \boldsymbol{\lambda}_{+}^{(\vartheta,i)})(\cdot)^{\mathrm{T}}$,

$t_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = t_{+}^{(\vartheta,i} + 2$,

$\boldsymbol{T}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \boldsymbol{A}^{(j+1)} + \mathrm{E}^{(j)}[\alpha]\,\boldsymbol{B}^{(j+1)} + \boldsymbol{T}_{+}^{(\vartheta,i)}$,

$\mathrm{E}^{(j+1)}[\boldsymbol{P}^{-1}] = \frac{t_{Z,i}^{(\vartheta,\theta)^{(j+1)}} - n_{\boldsymbol{x}} - 1}{\boldsymbol{T}_{Z,i}^{(\vartheta,\theta)^{(j+1)}}}$,

$u_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = u_{+}^{(\vartheta,i)} + 2$,

$\boldsymbol{U}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \boldsymbol{C}^{(j+1)} + \mathrm{E}^{(j)}[\beta]\,\boldsymbol{D}^{(j+1)} + \boldsymbol{U}_{+}^{(\vartheta,i)}$,

$\mathrm{E}^{(j+1)}[\boldsymbol{R}^{-1}] = \frac{u_{Z,i}^{(\vartheta,\theta)^{(j+1)}} - n_{\boldsymbol{z}} - 1}{\boldsymbol{U}_{Z,i}^{(\vartheta,\theta)^{(j+1)}}}$,

$\boldsymbol{K}^{(j+1)} = \bar{\boldsymbol{P}}_{+}^{(j+1)}\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{H}\bar{\boldsymbol{P}}_{+}^{(j+1)}\boldsymbol{H}^{\mathrm{T}} + \bar{\boldsymbol{R}}^{(j+1)})^{-1}$,

$\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = \boldsymbol{s}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} + \boldsymbol{K}^{(i+1)}(\boldsymbol{z} - \boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} - \boldsymbol{H}\boldsymbol{s}_{Z,i}^{(\vartheta,\theta)^{(j+1)}})$,

$\boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = (\boldsymbol{I}_{n_{\boldsymbol{x}}} - \boldsymbol{K}^{(i+1)}\boldsymbol{H})\bar{\boldsymbol{P}}_{+}^{(j+1)}$,

$a_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = a_{+}^{(\vartheta,i)} + 0.5n_{\boldsymbol{x}}$,

$b_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = b_{+}^{(\vartheta,i)} + 0.5\mathrm{tr}\left\{\boldsymbol{B}^{(j+1)}\mathrm{E}^{(j+1)}\left[\boldsymbol{P}_{+}^{-1}\right]\right\}$,

$\mathrm{E}^{(j+1)}[\alpha] = \frac{a_{Z,i}^{(\vartheta,\theta)^{(j+1)}}}{b_{Z,i}^{(\vartheta,\theta)^{(j+1)}}}$,

$c_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = c_{+}^{(\vartheta,i)} + 0.5n_{\boldsymbol{z}} - 1$,

$d_{Z,i}^{(\vartheta,\theta)^{(j+1)}} = 0.5\mathrm{tr}\left\{\boldsymbol{D}^{(j+1)}\mathrm{E}^{(j+1)}\left[\boldsymbol{R}^{-1}\right]\right\} + d_{+}^{(\vartheta,i)}$,

$\mathrm{E}^{(i+1)}[\beta] = \frac{c_{Z,i}^{(\vartheta,\theta)^{(j+1)}}}{d_{Z,i}^{(\vartheta,\theta)^{(j+1)}}}$,

end for

$\boldsymbol{m}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{m}_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $\boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{\Sigma}_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $\boldsymbol{s}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{s}_{Z,i}^{(\vartheta,\theta)^{(N)}}$,
$\boldsymbol{S}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{S}_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $t_{Z,i}^{(\vartheta,\theta)} = t_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $\boldsymbol{T}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{T}_{Z,i}^{(\vartheta,\theta)^{(N)}}$,
$\boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{\lambda}_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $\boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{\Lambda}_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $\boldsymbol{u}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{u}_{Z,i}^{(\vartheta,\theta)^{(N)}}$,
$\boldsymbol{U}_{Z,i}^{(\vartheta,\theta)} = \boldsymbol{U}_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $a_{Z,i}^{(\vartheta,\theta)} = a_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $b_{Z,i}^{(\vartheta,\theta)} = b_{Z,i}^{(\vartheta,\theta)^{(N)}}$,
$c_{Z,i}^{(\vartheta,\theta)} = c_{Z,i}^{(\vartheta,\theta)^{(N)}}$, $d_{Z,i}^{(\vartheta,\theta)} = d_{Z,i}^{(\vartheta,\theta)^{(N)}}$

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where $\mathbf{I}_2$ denotes 3×3 identity matrix, $\otimes$ is the Kronecker product, and $T = 1$s is the sampling interval. The true process noise covariance matrix and mean vector are formulated as

$$Q_k = \begin{cases} {Q_0}^*, & \text{if } k < k_p \\ 4{Q_0}^*, & \text{else} \end{cases} \tag{48}$$

$$q_k = \begin{cases} {q_0}^*, & \text{if } k < k_p \\ 5{q_0}^*, & \text{else} \end{cases} \tag{49}$$

where $k_p = 31$s represents the time step parameters of the true process noise covariance matrix and mean vector change, and $\boldsymbol{q}_0^* = [1,1,1,1]^{\mathrm{T}}$ and ${Q_0}^* = \sigma_w \mathbf{I}_2 \otimes \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix}$ represents the nominal value of process noise mean vector and covariance matrix respectively, and the standard deviation $\sigma_w = 5m/s^2$.

The measurement model can be described as (2), $\boldsymbol{H}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, and the true measurement noise covariance matrix and mean vector are given by

$$\boldsymbol{R}_k = \begin{cases} {\boldsymbol{R}_0}^*, & \text{if} k < k_m \\ 4{\boldsymbol{R}_0}^*, & \text{else} \end{cases} \tag{50}$$

$$\boldsymbol{r}_k = \begin{cases} {\boldsymbol{r}_0}^*, & \text{if} k < k_m \\ 5{\boldsymbol{r}_0}^*, & \text{else} \end{cases} \tag{51}$$

where $k_m = 51$s represents the time step parameters of the true measurement noise covariance matrix and mean vector change, and ${\boldsymbol{r}_0}^* = [1,1]^{\mathrm{T}}$ and ${\boldsymbol{R}_0}^* = \sigma_v^2 \boldsymbol{I}_2$ represents the nominal value of measurement noise mean vector and covariance matrix respectively, and the standard deviation $\sigma_w = 5m$.

Clutter is modeled as a Poisson RFS with intensity $\boldsymbol{\kappa}(\boldsymbol{z}) = \lambda_c V u(\boldsymbol{z})$, where $\lambda_c = 2.5 \times 10^{-6} m^{-2}$ denotes the average clutter intensity, $V = 4 \times 10^6 m^2$ is the volume of the surveillance region (giving an average of 10 clutter per scan). The tuning parameter $\tau = 5$, forgetting factor $\rho = 0.99$, and the number of iterations $N_{\max} = 5$.

The birth model is an LMB RFS with parameters $\pi_{\Gamma,k}(\boldsymbol{\zeta}) = \{{\boldsymbol{r}_k}^{(i)}, {\boldsymbol{p}_{\Gamma,k}}^{(i)}(\boldsymbol{\zeta})\}_{i=1}^{(4)}$, where ${\boldsymbol{r}_k}^{(i)} = 0.03$ and

$$\boldsymbol{p}_{\Gamma,k}^{(i)}(\boldsymbol{\zeta}) = \mathcal{NNIWNIWGG}(\boldsymbol{\zeta}; \boldsymbol{\phi}_{\Gamma,k}^{(i)}) \tag{52}$$

where $\boldsymbol{\phi}_{\Gamma,k}^{(i)} = (\boldsymbol{m}_{\Gamma,k}^{(i)}, \boldsymbol{\Sigma}_{\Gamma,k}, \boldsymbol{s}_{\Gamma,k}, \boldsymbol{S}_{\Gamma,k}, t_{\Gamma,k}, \boldsymbol{T}_{\Gamma,k}, \boldsymbol{\lambda}_{\Gamma,k}, \boldsymbol{\Lambda}_{\Gamma,k}, u_{\Gamma,k}, \boldsymbol{U}_{\Gamma,k}, a_{\Gamma,k}, b_{\Gamma,k}, c_{\Gamma,k}, d_{\Gamma,k})$ with $\boldsymbol{m}_{\Gamma,k}^{(1)} = [0.1, 0, 0.1, 0]^{\mathrm{T}}$, $\boldsymbol{m}_{\Gamma,k}^{(2)} = [400, 0, -600, 0]^{\mathrm{T}}$, $\boldsymbol{m}_{\Gamma,k}^{(3)} = [-800, 0, -200, 0]^{\mathrm{T}}$, $\boldsymbol{m}_{\Gamma,k}^{(4)} = [-200, 0, 800, 0]^{\mathrm{T}}$, $\boldsymbol{\Sigma}_{\Gamma,k} = \mathrm{diag}\left([10,10,10,10]^{\mathrm{T}}\right)^2$, $\boldsymbol{s}_{\Gamma,k} = [100, 0, 100, 0]^{\mathrm{T}}$, $\boldsymbol{S}_{\Gamma,k} = \mathrm{diag}\left([20,20,20,20]^{\mathrm{T}}\right)^2$, $t_{\Gamma,k} = 10$, $\boldsymbol{T}_{\Gamma,k} = 9\boldsymbol{Q}_0^*$, $\boldsymbol{\lambda}_{\Gamma,k} = [1,1]^{\mathrm{T}}$, $\boldsymbol{\Lambda}_{\Gamma,k} = \mathrm{diag}\left([10,10]^{\mathrm{T}}\right)^2$, $u_{\Gamma,k} = 20$, $\boldsymbol{U}_{\Gamma,k} = 16\boldsymbol{R}_0^*$, $a_{\Gamma,k} = 1$, $b_{\Gamma,k} = 1$, $c_{\Gamma,k} = 1$ and $d_{\Gamma,k} = 1$.

To further verify the effectiveness of the proposed NNIWNIWGG-δ-GLMB filter, it is compared with other filters over 100 Monte Carlo (MC) trials in the same simulation conditions. Fig.3 shows the cardinality statistics versus time for different filters. It can be seen that the GM-δ-GLMB filter with nominal noise mean vectors and covariance matrices, which is called GM-δ-GLMB (Nominal) for short, can correctly estimate the target number in the initial stage, but the target loss occurs from the 31st second, and further, from the

51st second, the target loss is more serious. Similarly, the performance of the NIWIW-δ-GLMB filter can still estimate the target number correctly at the 31st second. However, the target loss of the filter occurs from the 51st second, and becomes more obvious with the change of time. The target number estimation of the proposed NNIWNIWGG-δ-GLMB filter until the end of tracking is very close to that of the GM-δ-GLMB filter with true noise mean vectors and covariance matrices, which is called GM-δ-GLMB (True) for short.

Fig.4 shows the OSPA distance versus time over 100 MC trials. It is noted that the GM-δ-GLMB (True) filter and the NNIWNIWGG-δ-GLMB filter significantly outperform the NIWIW-δ-GLMB filter and the GM-δ-GLMB (Nominal) filter on the overall miss distance, and the GM-δ-GLMB (True) filter has a better performance than that of the proposed NNIWNIWGG-δ-GLMB filter.

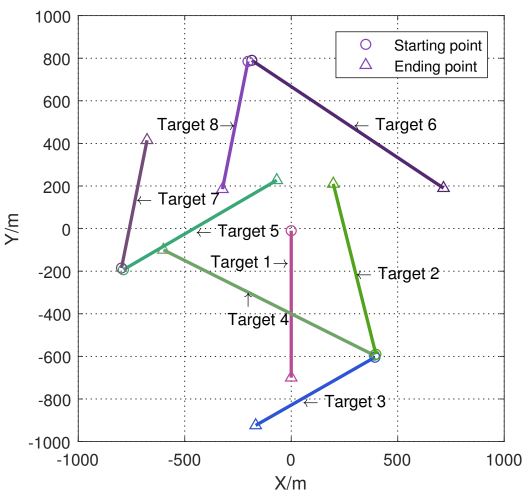

Fig. 2. True target trajectories.

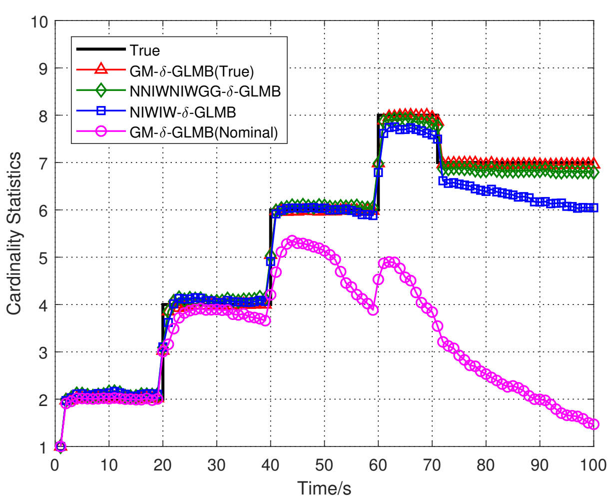

Fig. 3. Cardinality statistics for different filters.

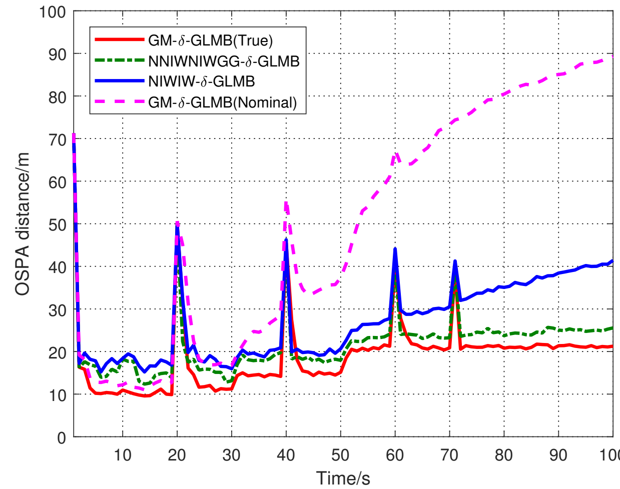

Fig. 4. Average OSPA distances for different filters.

VI. Conclusion

In this paper, an adaptive δ-GLMB filter based on VB inference for MTT under noise statistics mismatch was proposed, where the predicted state and the corresponding prediction error covariance matrix were modeled as an NIW distribution, and the latent parameter was modeled as a Gamma distribution. Similarly, the measurement noise mean vector and covariance matrix were also modeled as an NIW distribution, in which the latent parameter was also modeled as a Gamma distribution. Then, the target state, unknown parameters and latent parameters were modeled as augmented states. Then the joint single-target posterior density was described as mixtures of the NNIWNIWGG model, and the approximate posterior density of each parameter was derived by VB method. Finally, the approximation of the predicted likelihood was obtained by minimizing the Kullback-Leibler divergence. Simulation results showed that the tracking accuracy of the proposed NNIWNIWGG-δ-GLMB filter was comparable to that of the GM-δ-GLMB (True) filter.

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